

# OPTIMA

MATHEMATICAL PROGRAMMING SOCIETY NEWSLETTER

JUNE 1980  
NUMBER 1

Philip Wolfe. . . .

## The Ellipsoid Algorithm

Who would have thought that what must be the only front-page article about mathematics ever to appear in the *New York Times* ('A Soviet discovery rocks world of mathematics', November 7, 1979) would be about mathematical programming? Our Society was keeping up with Khachian before the *Times* made his work so famous. Some of us first heard of him at a Conference on Mathematical Programming in Oberwolfach, West Germany in May, 1979, where his Russian paper [1], really a long abstract, was circulated, but all we understood was its intriguing title. Eugene Lawler stirred up interest in it on his return to Berkeley, and by July Gacs and Lovasz, visiting Stanford University, had reconstructed the derivations of assertions related to Khachian's. At our invitation they presented their work to some 300 people at a special session of the Tenth International Symposium on Mathematical Programming held in Montreal in August. The science press then took notice, especially in an article [2] in *Science*, the version of the story which the *Times* further garbled, creating the impression that  $P = NP$  and that all the hard problems had been solved. Lawler has written an engaging account [3] of that part of the story. A flood of technical papers followed, which I have been trying to collect. Our bibliography [4] lists 46, as well as background literature, mostly Soviet, and some of the popular commentary. Now the phenomenon has peaked. The numbers of technical papers in each month (date on paper, if given; otherwise of receipt) are:

1979		1980	
Aug	1	Jan	14
Sep	1	Feb	8
Oct	1	Mar	5
Nov	9	Apr	1
Dec	6	May	0

This February the Society held a 'Workshop on polynomial-time algorithms for linear programming' attended by 80 people with an active interest in the subject. The 17 reports presented and lively discussions brought us pretty well up to date, and it appears that few new ideas have arisen since then. There are no written proceedings, but most of the short presentations were extracted from authors' longer written papers reviewed in [4].

The algorithm has a history. Most of it is actually due to other Soviet mathematicians: D.B. Yudin and A.S. Nemirovsky of Moscow, and N.Z. Shor of Kiev, whose article [5] states the algorithm in its clearest form, and also in its best form for practical computation -- not only for linear programming, but for the much more general convex programming problem. Because of the multiple authorship and its basic idea -- the generation of numbers which, geometrically, describe a sequence of ellipsoids which must all contain a solution and which shrink, so that it is eventually identified -- we refer to it and its near relatives as the 'ellipsoid algorithm'.

Actually, at least three different algorithms have been presented:

1. That to be attributed to Shor, Yudin, and Nemirovsky (above).
2. Khachian's [1]: This is Version 1, slightly modified so as to be provably implementable in polynomial time on a possible computer, but hardly practical.
3. That of Gacs and Lovasz [6]. While inspired by Khachian's, it uses different formulas. It is better viewed as a revision of Shor's algorithm, to which it is mathematically equivalent, although it is numerically ill-behaved. The ellipsoid becomes an hyperboloid after a few hundred iterations, destroying the computation. Many authors use the term

See page 3

ABOUT. . . .

## OPTIMA

Lately the Mathematical Programming Society has been trying to broaden its activities in a number of ways. Phil Wolfe's report to the membership distributed in Montreal last August testifies to this fact. In order to continue and enhance these efforts many of the officers and members of the society felt that it would be desirable to have an informal means of communication available and that this could best be attained by establishing a Newsletter. In addition it has been pointed out that a Newsletter could also be used as a vehicle for an outreach program on the part of the Society by, for example, permitting a new class of membership for students.

Consequently the Publications Committee began to explore several possibilities. Recognizing that the SIGMAP Newsletter (now Bulletin) served the mathematical programming community well in the past, one proposal was to found a joint Newsletter. Both SIGMAP and the ACM were somewhat receptive but not overly enthusiastic about such a venture. However, of much greater significance was the fact that when such a proposal was aired in Montreal in meetings of the Publications Committee and the Council, it became evident that a significant sentiment existed in favor of establishing our own Newsletter. Another proposal that was explored was the possibility of expanding the COAL Newsletter. However, it quickly became evident that this would not be feasible because of the highly specialized interest of COAL and the ad hoc nature of its publication.

Thus the decision was made to establish a new Newsletter--OPTIMA. The name was suggested by Phil Wolfe since it translates so well into many languages. A successful search to find an Editor culminated in the appointment of Don Hearn to this important position. We are indeed fortunate that such an able and energetic person as Don has agreed to undertake this task. I know that I speak for all members of the Society in wishing him every success in the launching of OPTIMA. However OPTIMA

See page 2

OPTIMA. . . .from page 1

will only succeed if YOU, the members, support it through your submissions and suggestions. Elsewhere in this first issue you will find a column by Don stating the kind of material he is seeking.

Many people worked hard to bring OPTIMA into being. In particular I would like to thank George Nemhauser who was always there when needed and who also nominated the successful candidate for Editor. Thanks are also due to Mike Powell who debated most persuasively during the many discussions which were held.

—Michael Held  
Chairman  
Publications Committee  
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DEADLINES FOR OPTIMA

- Fall Issue - September 1
- Winter Issue - December 1

Workshop

The IV Bonn Workshop on Combinatorial Optimization will take place August 28-30, 1980, at the Institute für Ökonometrie und Operations Research, University of Bonn.

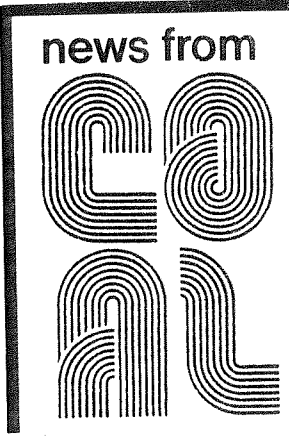
As with the preceding ones, the workshop is devoted primarily to recent research in the area of discrete and combinatorial optimization and related topics such as graph theory, matroids and independence systems, polyhedral combinatorics, analysis of combinatorial algorithms, etc. In addition to the members of the Institute and its visitors for the academic year 1980/81, leading experts of these fields will participate.

The structure of this workshop will be very informal. There will be no program in advance. The time schedule for the lectures will be made on the spot, but participants are expected to give their most recent and very best research papers in the above mentioned areas.

If there is enough interest we might arrange a hike in the Eifel for Sunday, August 31, or a boat trip on the River Rhine.

Participants are kindly requested to notify us of their intention to attend at their earliest convenience. Further information can be obtained from the Institute.

—Bernhard Korte



Computational results are often used in evaluating mathematical programming algorithms. To date, our profession has not developed a clear understanding of how computational testing should be carried out.

The Committee on Algorithms (COAL) was formed with the major objectives of (1) developing procedures for appraising the usefulness of computational results; (2) encouraging those who distribute programs to meet certain standards of portability, testing, ease of use, and documentation; and (3) acting as a focal point for information about computer programs that are available for general calculations and for test problems and test problem generators.

Since 1973, when COAL was formed, its members have been active in many areas including the collection of test problems, the investigation of techniques for removing timing, compiler, and computer variability from computational results, the establishment of reliable performance evaluation criteria for comparing mathematical programming software, and the development of a sound methodology for comparing MP software. Guidelines for reporting results of computational experiments were developed by three members of COAL and published in the following journals: Mathematical Programming, JORSA, and Transactions on Mathematical Software.

COAL has organized sessions presenting results of software testing research at conferences sponsored by the Mathematical Programming Society, IFAC, The European Congress on Operations Research, the Institute for Management Science, and the Operations Research Society of America. In 1978, a NATO Advanced Study Research Institute entitled "Design and Implementation of Optimization Software" was organized by COAL and a text of the proceedings, edited by Dr. Harvey Greenberg, is available through Sijthoff and Noordhoff.

Our work continues. In the near

future, you will be receiving a questionnaire in the mail from Dr. Kenneth Ragsdell regarding the availability of software for solving NLP problems. This is COAL's first attempt to collect a catalogue of available MP software. The catalogue will contain the following information: the code name, the method(s) used, special features of the code, references to the literature, the code's author(s), distribution source for the code, and the conditions of availability. We hope to conduct similar surveys in other fields of math programming in the future.

At the EURO IV meeting to be held July 22-25, 1980 in Cambridge, England, COAL will sponsor a session chaired by Dr. Susan Powell in which the following talks will be presented:

- "Mathematical Programming Systems—Master or Servant?," E.M. Beale
- "The Relationship Between O.R. and Software," M. Jeffreys
- "Performance Evaluation of Nonlinear Programming Codes via Multi-criteria Decision Analysis," F. Lootsma
- "The Testing and Evaluation of Mathematical Programming Software," K. Hoffman and R. Jackson.

There will also be a session sponsored by COAL at the ORSA/TIMS meeting in Colorado Springs in November 1980.

On January 5-6, 1981, a conference on Testing and Validating MP Algorithms and Software will be held in Boulder, Colorado. This conference will bring together researchers from a variety of disciplines who have performed computational analysis (either in math programming or general computations) in order to further develop methodologies for conducting software evaluation.

Finally, the committee on Algorithms publishes a Newsletter twice a year. The purposes of this newsletter are to generate an international interchange of ideas, highlight research being performed in software testing, announce the availability of new software, and highlight international meetings where research on software evaluation will be presented.

The committee on Algorithms is a rather small group with extremely ambitious goals. Only with the assistance of much of the MP community can we succeed. Please help us by answering our surveys, contributing to our newsletter, and providing us with your suggestions as to how this Committee can better serve you.

—Karla Hoffman, Editor  
Committee on Algorithms Newsletter

'Khachian's algorithm' while actually working with this version. Those who have been able to compute successfully have used some form of Version 1.

One obtains the versions above by setting  $d = 0$  in equations (3,4) below. The ellipsoid (1) is then 'cut' through its center by intersecting it with the halfspace  $\{y : a \cdot (y - x) \leq 0\}$ , whose boundary is parallel to the chosen violated constraint, and the new ellipsoid, which is that of smallest volume circumscribing the intersection, is given by the updated quantities. Using (2) instead to calculate  $d$  gives a cut whose boundary is the violated constraint; the volume of the resulting ellipsoid may be much smaller. (It is remarkable that none of the Soviet authors gave the formula for the 'deep cut', which is not hard to work out. It was discovered independently by authors of 18 of our 46 technical papers.)

Recommended implementation of the 'ellipsoid' algorithm for solving  $Ax \leq b$ , where  $A$  is an  $m$  by  $n$  matrix and  $x \in \mathbb{R}^n$ :

*Start:* Have  $x$  and a nonsingular matrix  $J$  such that the ellipsoid

$$(1) \quad \{x : |J^{-1}(y - x)| \leq 1\}$$

will contain a solution of the system, if any exists.

*Recursion* ( $x, J$  are replaced by  $x_+, J_+$  below):

Find a violated inequality  $a \cdot x - \beta > 0$ .

$$(2) \quad \text{Set } d = \frac{a \cdot x - \beta}{|Ja|}$$

$$(3) \quad \text{Set } x_+ = x - \frac{1 + nd}{1 + n} \frac{J^T Ja}{|Ja|}$$

$$(4) \quad J_+ = n \sqrt{\frac{1-d^2}{n^2-1}} \left[ I - \left( 1 - \sqrt{\frac{(n-1)(1-d)}{(n+1)(1+d)}} \right) \frac{Ja(Ja)^T}{|Ja|^2} \right] J$$

It seems to be good practice to choose a substantially violated inequality, but not to do all the work required to choose one which maximizes  $d$ . If  $d > 1$ , then there is no solution.  $J^T$  is proportional to  $A_k$  of Shor and serves as the  $Q$  of Khachian.  $J^T J$  is the  $A_k$  of Gacs and Lovasz.

The basic theory of the method is that each ellipsoid contains all the feasible points of its predecessor, while the ratio of its volume to that of the predecessor is

$$(5) \quad n^n \left( \frac{1-d^2}{n^2-1} \right)^{n/2} \sqrt{\frac{(n-1)(1-d)}{(n+1)(1+d)}}$$

For  $d = 0$  this ratio never exceeds  $e^{-1/2n}$  (and is asymptotically equal to that quantity for large  $n$ ) so the ellipsoids steadily shrink. If the feasible set has a positive volume then the algorithm must stop (with a new  $x$  violating no inequality), while if it has zero volume then the centers  $x$  must converge toward the feasible set (although not with any particular monotonicity). If the feasible region is empty, then one should eventually discover  $d > 1$ .

All that is mathematics, of course, and we know that direct transcription of formulas into computer code, in the hope that the computer's representation of mathematic's 'real' numbers will be accurate enough to make the theory work, has pitfalls.

Khachian's formulas differ from those above in only one important respect: he further multiplies  $J$  by a factor slightly greater than one to ensure that the new ellipsoid will indeed enclose the old half-ellipsoid. In order to do

See page 4

# The Newsletter

OPTIMA's primary purpose is to be a forum for the membership, providing information about current research, a calendar of meetings, news about members, etc.

In addition to these regular items, we plan to feature at least one article of broad interest in each issue. Examples of sources are plenary addresses, articles about the early days of mathematical programming, and articles about special research activities. When a rare technical article appears that can be called "news", it could be a feature article. Usually, however, we will be looking for articles which offer perspective on the field of mathematical programming, its history and its directions. Such articles will be edited, as will all contents, but they will not be refereed.

Anyone wishing to contribute a feature article should contact me so that we can make plans. The newsletter will vary in size from 8 to 16 pages and as much as one-third to one-half that amount could be allocated to an article.

The departments which will appear on a reasonably regular basis are:

- Chairman's Column
- Conference Calendar
- Technical Report Titles
- Letters to the Editor
- News from COAL
- Brief News Items

Contributions to these, announcements of meetings, plus any other articles of interest are invited.

Some space will be available for advertising position announcements and short courses. The fee, for three column inches (150-200 words) will be fifty (U.S.) dollars per insertion. Submission of such announcements should be accompanied by a check in this amount payable to OPTIMA. Other advertising, from publishers, for example, can also be accepted.

Any comments or suggestions about OPTIMA are, of course, welcome.

—Don Hearn

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## OPTIMA

Newsletter of the Mathematical Programming Society

Donald W. Hearn, Editor

Judy C. Tolbert, Editorial Assistant

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ALGORITHM. . . .from page 3

that using finite precision, he uses the factor  $2^{1.8n^2}$  and supposes that his computer can accommodate fixed-point data having  $23L$  bits to the left and  $38nL$  bits to the right of the binary point, where  $L$  is the total number of bits required to encode the data of the particular problem. He then notes that (i) if the system has a solution, then it has one in a sphere of radius  $2^L$  (so that  $x = 0, J = 2^L I$  works in (1) above), and (ii) if the system has a solution, and its inequalities are relaxed by the quantity  $2^{-L}$ , then the feasible set has volume at least  $2^{-L}$ . With his approximations he can work down from the starting volume to this final volume in no more than  $16n^2L$  steps, resolving the question as to whether or not the system is feasible. Since this number, as well as the number of arithmetic operations per step required ( $O(n^3)$ ) is polynomial in the length  $L$  of the input string, so is the time for the whole algorithm.

Of course, Khachian's short paper does not give the proofs that all this works; indeed, it does not show just how to find a feasible point if there is one, let alone how to solve a linear programming problem. A number of authors have filled in the latter gap, and Padberg and Rao [7] give the full proofs.

In practice the formulas (1-4) do work on a real computer. I compute using APL on an IBM 370/168. The arithmetic is 370 double precision: 54-56 bits mantissa, exponent range  $10^{\pm 77}$ . I have had no trouble on various problems, taking up to as many as 30,000 steps: the mathematics seems to work, and when I compute the volume of an ellipsoid, it has (in the  $d = 0$  case) very nearly the predicted volume. I have also tried some small problems in which the feasible region was a single point, and had 'trouble' (signalled by a false indication of infeasibility) only after the procedure had produced a point that the computer could hardly distinguish from the right answer.

Perhaps one reason the Russian authors have wisely not offered the ellipsoid algorithm as an alternative to the simplex method is that the theory, at least for the  $d = 0$  case, predicts disarmingly slow convergence. In order to add one decimal place to the accuracy of a solution -- i.e., to reduce the distance to a solution by a factor of 10 in each dimension -- we can expect to have to reduce the volume of the current ellipsoid by a factor of  $10^{-n}$ . Using the estimate above for the ratio (5), in  $k$  steps the volume will be reduced by the factor  $e^{-k/2n}$ ,

which gives  $k = 4.6n^2$  as the required number. We would expect the simplex method to solve such a problem nearly to machine accuracy in less than  $2n$  steps.

The following data are typical. The illustrative problem in the user's manual for the IBM linear programming routine MPSX/370 has 7 variables, 14 inequalities, and 1 equation (which I write as 2 inequalities). Writing the primal and dual together (admittedly not the best way to handle the problem), I get a system of 47 inequalities in 23 variables. I chose the initial ellipsoid as a sphere of radius 1000 about the origin. My program monitors the worst violation of any inequality; the table below gives the number of the first step on which the worst violation fell below the indicated power of ten (after being only 200 initially).

Exp.	Iteration	
	Version 1	Deep Cut
5	2	2
4	4	4
3	12	6
2	17	57
1	934	586
0	2941	1157
-1	5168	1823
-2	7218	2545
-3	9280	3368
-4	11493	4103
-5	13820	4806
-6	16130	5618
-7	18367	6328

# George B. Dantzig Prize

The Society, jointly with the Society of Industrial and Applied Mathematics, will sponsor The George B. Dantzig Prize in honor of Professor Dantzig's contributions to mathematical programming. First award of the prize is anticipated at the Eleventh Mathematical Programming Symposium in Bonn in 1982. The prize is scheduled for possible award every three years thereafter, with every third award being at a SIAM national meeting.

The award, in the form of a certificate containing the citation and a cash prize, is to be made for original work, which by its breadth and scope, constitutes an outstanding contribution to the field. Nominations will be made to the Societies' Executive Committees by an ad hoc committee of MPS and SIAM members. Committee Chairman for the first award is Professor Roger J. B. Wets of the University of Kentucky, who, along with Richard Cottle, Ellis Johnson and Richard van Slyke, suggested the Dantzig

Prize. All are former students of Professor Dantzig.

The prize specifications state that the contribution(s) for which the award is made must be publicly available and that they may belong to any aspect of mathematical programming in its broadest sense. The contribution(s) eligible for consideration are not restricted with respect to the age or number of their authors although preference will be given to singly-authored work of "younger" people. Professor Wets has stated that the committee will seek work in the spirit of Dantzig's own in that, ideally, it should contribute to both application and theory.

Support for the Dantzig prize will come from individual, corporate, and institutional contributions. Those wishing to contribute should send a check payable to the G.B. Dantzig Fund to SIAM, 33 S. 17th Street, Philadelphia, Pa. 19103. Individual contributions are tax deductible.

The number of steps to reduce by 0.1 is close to the predicted value 2433 for Version 1, while the deep cut speeds things up by a factor of 3 -- amounting to a trifling reduction in the ellipsoid volumes.

Our own APL version of the simplex method required 21 iterations to solve the 47-inequality problem (reduced to a 24-inequality problem in nonnegative variables), with a maximum violation of  $10^{-12}$ . It takes 10 or 15 iterations to solve the original linear programming problem, depending on whether or not the usual special device for handling upper-bounded variables is used.

There are still, of course, a great number of opportunities for further improvements, and many have been suggested in the papers received. Besides the deep cut, improvements proposed for the basic

## ALGORITHM. . . .from page 4

algorithm include: general methods for getting a smaller starting ellipsoid; discarding unneeded inequalities and early determination of infeasibility; combining inequalities for improved cuts; and ways for removing equations from the system. Further, possibly better ways to do linear programming than by the expensive simultaneous primal-dual formulation are proposed: hybridization with simplex or related methods; the construction of cuts using the objective function; and using complementary slackness to predict irrelevant variables and constraints so that the size of the system can be progressively reduced. (The references 8-10 cover most of these points.)

Despite these opportunities, we have yet to hear of evidence that the ellipsoid algorithm can compete with the simplex method. Since the invention of the latter in 1947, mathematical improvements in the algorithm itself have probably speeded it up by no more than a factor of 10. The most important development has been in data-handling -- the exploitation of sparseness, which accounts for several orders of magnitude in computing time for problems of serious size. We think the ellipsoid algorithm would not have competed with the simplex method if it had been available in 1947; further, no one seems to have found a solid way to exploit sparseness in it.

However, whatever its demerits for linear programming, we can say this for the ellipsoid algorithm: it solved a significant theoretical problem, and can be used to solve others; it may still be practically useful for the difficult nonlinear problems that Shor proposed it for; and it certainly brought a new kind of excitement to our area of applied mathematics.

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7. M.W. Padberg and M.R. Rao, 'The Russian method for inequalities II: approximate arithmetic', January 1980. Graduate School of Business Administration, New York University, New York, NY 10006, U.S.A.
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9. P.C. Jones and E.S. Marwil, 'A dimensional reduction variant of Khachian's algorithm for linear programming problems', January 1980. EG & G Idaho, Inc., P.O.B. 1625, Idaho Falls, Idaho 83415, U.S.A..
10. P.F. Pickel, 'Some improvements to Khachian's algorithm in linear programming', December 1979. Polytechnic Institute of New York, Route 110, Farmingdale, N.Y. 11735, U.S.A.



### International Summer School in Optimization Techniques and Applications

A summer school entitled "Optimization: Techniques and Applications" will be held June 30 -- July 11, 1980 at the Balls Park Site of the Hatfield Polytechnic. Director for the course is Dr. L.C.W. Dixon.

Course fees are £ 200 for registration and £ 150 for accomodations. For further information, contact Mrs. P. Ingram, School of Information Sciences, The Hatfield Polytechnic, P.O. Box 109, Hatfield, Herts. AL10 9AB, United Kingdom.

### 1980 N.A.T.O. Advanced Research Institute on Generalized Concavity

A NATO Advanced Study Institute on "Generalized Concavity in Optimization and Economics" will be held in Vancouver/Canada on August 4-15, 1980. The school is directed by M. Avriel, Haifa; S. Schaible, Edmonton; and W.T. Ziemba, Vancouver. Topics will include characterizations of various concepts of generalized concavity, special functional forms, optimality and duality, fractional programming, multicriteria optimization, numerical solution methods, applications in management science, statistics and economics. The number of participants is limited to 80. Those wishing to participate should contact Professor Siegfried Schaible, Faculty of Business Administration, University of Alberta, Edmonton, Alberta T6G 2G1, Canada, as soon as possible.

--S. Schaible

### 1981 N.A.T.O. Advanced Research Institute on Nonlinear Optimization

A NATO Advanced Research Institute on Nonlinear Optimization (co-sponsored by the Mathematical Programming Society) will be held in Cambridge, England from July 13th - 24th, 1981. Because the main purpose of these Institutes is to make a critical assessment of the current knowledge of a subject, and to publish conclusions for the benefit of a wider community, the main part of the program will be discussion sessions on different subjects in Nonlinear Optimization, including the theory and development of algorithms and their software. Key addresses at the discussions will be given by E.M.L. Beale, J.E. Dennis, R. Fletcher, P.E. Gill, F. Lootsma, J.J. Moré and R.B. Schnabel. Also there will be many opportunities for informal discussions and for participants to present seminars on their recent research. Because the number of participants is limited to about fifty, attendance at the Institute is by invitation only, and the organizing committee will meet in January 1981 to select participants. In order that the committee can give proper consideration to those people who wish to attend, you are invited to submit an application if you would like to participate. Application forms and further information are available from Professor M.J.D. Powell, DAMTP, Silver Street, Cambridge CB3 9EW, England.

--M.J.D. Powell

# Technical Reports & Working Papers

UNIVERSITY OF BONN  
Department of Operations Research  
D-5300 Bonn, West Germany

- R. Giles, "Adjacency on the postman polyhedron," WP 79128-OR.  
M. Grötschel, F. Harary, "The graphs for which all strong orientations are Hamiltonian," WP 79129-OR.  
A. Bachem, B. Korte, "Minimum norm problems over transportation polytopes," WP 79130-OR.  
A. Bachem, "Theorems of the alternative in combinatorial programming," WP 79131-OR.  
D. Hausmann, R. Kannan, B. Korte, "Exponentielle untere Komplexitätsschranken für eine Klasse von Knapsack-Problemen," WP 79132-OR.  
D. Hausmann, "Farbklassen und Pseudoknoten für Adjazenzcharakterisierungen," WP 79133-OR.  
R. W. Cottle, "Observations on a class of nasty linear complementarity problems," WP 79134-OR.  
M. Grötschel, C. Thomassen, Y. Wakabayashi, "Hypotractable digraphs," WP 79135-OR.  
R. Giles, L. Trotter, "On stable set polyhedra for  $K_{1,3}$ -free graphs," WP 79136-OR.  
A. Bachem, R. Schrader, "Minimal inequalities and subadditive duality," WP 79137-OR.  
R. Cottle, R. von Randow, "On  $Q$ -matrices, centroids and simplices," WP 79138-OR.  
C.L. Monma, L.E. Trotter, Jr., "On perfect graphs and polyhedra with  $(0,1)$ -valued extreme points," WP 79139-OR.  
D. Pallaschke, "An algebraic representation of classical systems," WP 79140-OR.  
D. Hausmann, B. Korte, "Algorithmic versus axiomatic definitions of matroids," WP 79141-OR.  
A. Bachem, M. Grötschel, "Adjacency relations on polyhedra," WP 79142-OR.  
D. Hausmann, B. Korte, "The relative strength of oracles for independence systems," WP 79143-OR.  
D. Hausmann, B. Korte, "Computational relations between various definitions of matroids and independence systems," WP 79144-OR.  
"Sommerschule über Optimierung und Operations Research, Zusammenfassung der Vorträge," WP 79145-OR.  
A. Bachem, "Concepts of algorithmic computations," WP 79146-OR.  
M. Grötschel, "Approaches to hard combinatorial optimization problems," WP 79147-OR.  
B. Korte, "Matroids and independence systems," WP 79148-OR.  
A. Bachem, M. Grötschel, "Proof techniques in polyhedral theory," WP 79149-OR.

## UNIVERSITY OF CAMBRIDGE

Department of Applied Mathematics and Theoretical Physics  
Silver Street, Cambridge CB3 9EW, England

- M. J. D. Powell, "Quasi-Newton formulae for sparse second derivative matrices," DAMTP 1979/NA7.  
M. J. D. Powell, "Optimization algorithms in 1979," DAMTP 1979/NA9.  
R. M. Chamberlain, C. Lemarechal, H. C. Pederson and M. J. D. Powell, "The watchdog technique for forcing convergence in algorithms for constrained optimization," DAMTP 1980/NA1.

CLEVELAND STATE UNIVERSITY  
Computer Science Department  
Cleveland, Ohio 44115

- L.S. Ladsdon and A.D. Warren, "A Survey of Nonlinear Programming Applications," Wp 80-01.

UNIVERSITY OF FLORIDA  
Industrial and Systems Engineering  
303 Weil Hall  
Gainesville, FL 32610

- Russell R. Barton and Donald W. Hearn, "Decomposition Techniques for Nonlinear Cost Multicommodity Flow Problems," 79-2.  
B. C. Tansel, R.L. Francis, and T.J. Lowe, "Duality and the Nonlinear  $p$ -Center Problem and Covering Problem on a Tree Network," 79-3.  
Donald W. Hearn and James Jesunathadas, "Analysis and Extensions of Algorithms for Sylvester's Minimax Location Problem," TR 80-1.  
Luc G. Chahmet, "Efficiency in Minimum Rectilinear Distance Location Problems," TR 80-2.  
Donald W. Hearn and Jaime Ribera, "On Daganzo's Modified Frank-Wolfe Method for Certain Bounded Variable Traffic Assignment Problems," TR 80-3.  
Donald W. Hearn, "Bounding Flows in Traffic Assignment Models," TR 80-4.  
Barbaros C. Tansel, Richard L. Francis and Timothy J. Lowe, "A Bi-Objective Multifacility Minimax Location Problem on a Tree Network," TR 80-6.  
M.L. Chen, R. L. Francis, T.J. Lowe and B.C. Tansel, "Distance Constraints for Tree Network Versions of the Nonlinear  $p$ -Center and Covering Problems," TR 80-7.

## MATHEMATICS RESEARCH CENTER

University of Wisconsin  
610 Walnut Street  
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- O. L. Mangasarian, "Optimal simplex tableau characterization of unique and bounded solutions of linear programs," TR 2034, 1980.  
S. M. Robinson, "Some continuity properties of polyhedral multifunctions," TR 2014, 1979.

## McGILL UNIVERSITY

School of Computer Science  
805 Sherbrooke Street West  
Montreal, Quebec H3A 2K6

- V. Chvátal, "Hard Knapsack Problems," 79.1  
D. Avis, "On the Complexity of Finding the Convex Hull of a Set of Points," 79.2  
V. Chvátal, "Cheap, Middling and Dear," 79.3  
D. McCallum and D. Avis, "Finding the Convex Hull of a Simple Polygon," 79.5  
G. T. Toussaint, "The Relative Neighbourhood Graph of a Finite Planar Set," 79.7  
L. Devroye, "A Bibliography on Random Search," 79.9  
V. Chvátal, "Recognizing Intersection Patterns," 79.10  
D. Avis, "A Note on Some Computationally Difficult Set Covering Problems," 79.11  
R. Nigel Horspool, "Constructing the Voronoi Diagram in the Plane," 79.12  
H.T. Lau, "Finding EPS-Graphs," 70.15  
S. Kourouklis, "Computing Weighted Linear Least Squares," 79.16  
V. Chvátal and E. Szemerédi, "On the Erdos-Strong Theorem," 79.17  
J. Akiyama, D. Avis, V. Chvátal and H. Era, "Balancing Signed Graphs," 79.18  
M. Ajtai, V. Chvátal, M.M. Newborn and E. Szemerédi, "Crossing-free hamiltonian circuits," 79.21

PRINCETON UNIVERSITY  
School of Engineering and Applied Science  
Princeton, New Jersey 08544

Richard H. F. Jackson, John M. Mulvey, "A critical review of comparisons of mathematical programming algorithms and software (1953-1977)," EES 78-8.

John M. Mulvey, "Maximizing homogeneity for multivariate stratified sampling," ES 79-1.

John M. Mulvey, "Reducing micro-data files by optimization models or by stratified sampling," EES 79-2.

Fred Glover, John M. Mulvey, "Equivalence of the 0-1 integer programming problem to discrete generalized and pure networks," EES 79-5.

Jonathan S. H. Kornbluth, Ralph E. Steuer, "Multiple objective linear fractional programming," EES 79-7.

Fred Glover, John M. Mulvey, "Network relaxations and penalties for multiple choice problems," EES 79-12.

Jonathan S.H. Kornbluth, Ralph E. Steuer, "On computing the set of all weakly efficient vertices in multiple objective linear fractional programming," EES 79-13.

SYSTEMS OPTIMIZATION LABORATORY  
Department of Operations Research  
Stanford University  
Stanford, CA 94305

Eric Rosenberg, "Globally Convergent Algorithms for Convex Programming," SOL 79-1.

George B. Dantzig and Peter L. Jackson, "Pricing Underemployed Capacity in a Linear Economic Model," SOL 79-2.

S. C. Parikh, "A Welfare Equilibrium Model (WEM) of Energy Supply, Energy Demand, and Economic Growth," SOL 79-3.

Nathan Buras, "Determining the Feasibility of Integrating Water Resource Constraints into Energy Models," SOL 79-4.

Michael A. Saunders, "Sparse Least Squares by Conjugate Gradients: A Comparison Preconditioning Methods," SOL 79-5.

Frederick S. Hillier and Nancy Eileen Jacquin, "A Bounding Technique for Integer Linear Programming with Binary Variables," SOL 79-6.

Nancy Eileen Jacquin, "Documentation of a Computer Program for the Bound-and-Scan Algorithm for Integer Linear Programming," SOL 79-7.

Nancy Eileen Jacquin, "Documentation of a Computer Program for Hillier's Heuristic Procedure in Integer Linear Programming," SOL 79-8.

A.I. Simon and N. Buras, "Integration of Water Resource Constraints in Energy Models," SOL 79-9.

Richard W. Cottle and Rabe von Randow, "On Q-Matrices, Centroids and Simplexes," SOL 79-10.

Eric Rosenberg, "A Globally Convergent Condensation Method for Geometric Programming," SOL 79-11.

Richard W. Cottle, "Completely Q-Matrices," SOL 79-12.

Mukund Thapa, "A Note on Sparse Quasi-Newton Methods," SOL 79-13.

Yow-Yieh Chang, "Least-Index Resolution of Degeneracy in Linear Complementarity Problems," SOL 79-14.

Philip E. Gill and Walter Murray, "Conjugate-Gradient Methods for Large Scale Nonlinear Optimization," SOL 79-15.

Benjamin Avi-Itzhak and Alfredo Iusem, "A Consumers Energy Services Model," SOL 79-16.

Robert Fourer, "Sparse Gaussian Elimination of Staircase Linear Systems," SOL 79-17.

Robert Fourer, "Solving Staircase Linear Programs by the Simplex Method, 1: Inversion, and 2: Pricing," SOL 79-18 and SOL 79-19.

Richard K. McCord, "Minimization with One Linear Equality Constraint and Bounds on the Variables," SOL 79-20.

Walter Murray and Michael L. Overton, "A Projected Lagrangian Algorithm for Nonlinear Minimax Optimization," SOL 79-21.

George B. Dantzig, "Comments on Khachian's Algorithm for Linear Programming," SOL 79-22.

Jerome H. Friedman and Margaret H. Wright, "A Nested Partitioning Procedure for Numerical Multiple Integration," SOL 79-23.

Margaret H. Wright, "Algorithms for Nonlinearly Constrained Optimization," SOL 79-24.

P.E. Gill, W. Murray, M.A. Saunders, and M.H. Wright, "Two Step-Length Algorithms for Numerical Optimization," SOL 79-25.

Zachary F. Lansdowne, "Survey of Research on Model Aggregation and Simplification," SOL 79-26.

B. Avi-Itzhak, T.J. Connolly, and A. Iusem, "The Coal Module of the Pilot System," SOL 79-27.

R.A. Murtagh and M.A. Saunders, "The Implementation of a Lagrangian Based Algorithm for Sparse Nonlinear Constraints," SOL 80-1.

Walter Murray and Margaret H Wright, "Computational of the Search Direction in Constrained Optimization Algorithms," SOL 80-2.

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BEB 608  
Austin, TX 78712

Fred Glover, Gene Jones, David Karney, Darwin Klingman, and John Mote, "An Integrated Production, Distribution, and Inventory Planning System."

Richard Barr, Fred Glover, and Darwin Klingman, "A New Optimization Method for Large-Scale Fixed Charge Transportation Problems."

Fred Glover, Darwin Klingman, John Mote, and David Whitman, "Comprehensive Computer Evaluation and Enhancement of Maximum Flow Algorithms."

Roy Crum, Darwin Klingman, and Lee Travis, "Strategic Management of Multinational Companies: Network-Based Planning Systems."

Fred Glover, Darwin Klingman, John Mote, and David Whitman, "A Primal Simplex Variant for the Maximum Flow Problem."

Darwin Klingman and John Mote, "Solution Approaches for Network Flow Problems with Multiple Criteria."

Darwin Klingman and John Mote, "Generalized Network Approaches for Solving Least Absolute Value and Tchebycheff Regression Problems."

L.S. Lasdon and A.D. Waren, "Generalized Reduced Gradient Software for Linearly and Nonlinearly Constrained Problems."

WASHINGTON STATE UNIVERSITY  
Department of Mathematics  
Pullman, Washington 99164

R. Mifflin, "A modification and extension of Lemarechal's algorithm for nonsmooth minimization," TR 80-1.

UNIVERSITY OF WISCONSIN-MADISON  
Computer Sciences Department  
1210 West Dayton Street  
Madison, Wisconsin 53706

C. Y. Kao, R. R. Meyer, "Secant approximation methods for convex optimization," TR 352, 1979.

O. L. Mangasarian, "Iterative solution of linear programs," TR 327, 1979.

O. L. Mangasarian, "Locally unique solutions of quadratic programs, linear and nonlinear complementarity problems," TR 345, 1979.

O. L. Mangasarian, "Characterizations of bounded solutions of linear complementarity problems," TR 359, 1979.

R. R. Meyer, "Continuity properties of linear programs," TR 373, 1979.

R. R. Meyer, "Computational aspects of two-segment separable programming," TR 382, 1980.



# <sup>8</sup>Gallimaufry

The John von Neumann Prize was awarded at the May, 1980 ORSA/TIMS meeting in Washington, D.C., to Profs. David Gale, Harold W. Kuhn and A.W. Tucker for contributions to the theory of mathematical programming. . . Dick Cottle, new Editor-in-Chief of the Journal and Studies has announced three new Co-Editors—L.C.W. Dixon (Hatfield Polytechnic), Bernhard Korte (Bonn), and M.J. Todd (Cornell). There are 10 new Associate Editors—E.L. Allgower (Colorado State), R.G. Jeroslow (Georgia Tech), D.S. Johnson (Bell Labs.) L. Lovasz (Szeged), M.W. Padberg (New York University), W.R. Pulleybank (Calgary), K. Ritter (Stuttgart), R.W.H. Sargent (Imperial College), D.F. Shanno (Arizona), and L.E. Trotter, Jr. (Cornell). An article on the structural changes and Editorial Board operation will be in Vol. 19, No. 1. S.M. Robinson (Wisconsin) has accepted the editorship of *Mathematics of Operations Research*. . . The University of Florida Center for Econometrics & Decision Sciences initiated a distinguished seminar series in the past year. Initial lectures were given by Nicos Christofides (Imperial College), George Nemhauser (Cornell), and Stan Zions (SUNY, Buffalo). . . Jerome Kreuser is the new Editor of the SIGMAP Bulletin. . . The most recent issue of SIGMAP (No. 24, April 1980) contains 8 papers (from a special session at the April 1979, ORSA/TIMS meeting) on "Recent and Future Development of Math Programming Systems" presented by hardware & software representatives. . . Society membership rose 20% from March 1979 to March 1980, due largely to the membership drive at the Montreal meeting. There are now 485 total members, 279 from North America, 131 from Western Europe, 24 from Eastern Europe, 26 from Asia and Australia, 17 from Latin America and 5 from Africa.

This Spring Prof. Bela Martos visited Purdue's Krannert School . . . J. Stoer (Wurzburg) is visiting the National Bureau of Standards. . . J. Telgren (Erasmus) has been visiting the University of Tennessee and is to visit SUNY, Buffalo. . . K. Schittkowski (Wurzburg) has recently visited Argonne, University of Texas and NBS. . . B.T. Polyak (Moscow) will be visiting the U.S. in early summer. His schedule is being arranged by D.P. Bertsekas (M.I.T.). . . Egon Balas (Carnegie-Mellon) will be visiting the Mathematisches Institut, Universität Köln, beginning September 1.

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## Technion Announces 1980-81 Pinhas Naor Distinguished Fellowship

The Faculty of Industrial Engineering and Management at the Technion - Israel Institute of Technology, invites applications for the 1980-81 Pinhas Naor Distinguished Fellowship. Pinhas (Paul) Naor was the founder of the Faculty and Professor of Operations Research until his death in an airplane accident in December 1970.

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Applications and nominations should be sent, by July 15, 1980, to Professor Michael Rubimovitch, Dean, Faculty of Industrial Engineering and Management, Technion, Haifa, Israel.

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# CALENDAR

Maintained by the Mathematical Programming Society (MPS)

This Calendar lists meetings specializing in mathematical programming or one of its subfields in the general area of optimization and applications, whether or not the Society is involved in the meeting. (These meetings are not necessarily "open".) Any one knowing of a forthcoming meeting not listed here is urged to inform the Chairman of the Executive Committee of the Society, Dr. A. C. Williams, Computer Science Department, Mobil Oil Co. Technical Center, Box 1025, Princeton, New Jersey 08540, USA; telephone 609-737-3000, extension 2342.

Substantial portions of regular meetings of other societies such as SIAM, TIMS, and the many national OR societies are devoted to mathematical programming, and their schedules should be consulted.

1980

- June 2-6: "Workshop on Large-scale Linear Programming", International Institute for Applied Systems Analysis, Laxenburg, Austria. Contact: Dr. Markku Kallio, IIASA, 22361 Laxenburg, Austria. Cosponsored by Systems Optimization Laboratory, Stanford University, and the MPS.
- June 16-20: "Workshop in Numerical Methods for System Engineering Problems" in Lexington, Kentucky. Contact: Professor Roger J.-B. Wets, Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506; telephone 606-257-2836. Sponsored by the MPS.
- July 7-11: "Tutorial Conference on Practical Optimization" in Stanford, California. Contact: Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California 94305, U.S.A.
- July 14-16: "Nonlinear Programming Symposium 4" in Madison, Wisconsin. Contact: Professor Olvi Mangasarian, Computer Sciences Department, University of Wisconsin, 1210 West Dayton Street, Madison, Wisconsin 53706; telephone 608-262-1204. Sponsored by the MPS.
- July 22-25: "Fourth European Congress on Operations Research" in Cambridge, England. At least one session on software testing will be sponsored by the Committee on Algorithms of the MPS. For general information on the Congress contact Prof. J. P. Brans, University of Brussels, VUB/CSOO, Pleinlaan 2, B-1050 Brussels, Belgium; regarding the Committee contact Dr. Susan Powell, Datalogisk Institut, Kobenhavns Universitet, DK-2200 Kobenhavn N, Denmark.
- July 28-August 1: "International Workshop on Advances in Linear Optimization Algorithms and Software", Pisa, Italy. Contact: Dott. Claudio Sandi, IBM Italia - Centro Scientifico, via Santa Maria 67, 56100 Pisa, Italy; telephone 50 (= Pisa) 47383.
- August 4-15: "Generalized concavity in Optimization and Economics", NATO Advanced Study Institute, University of British Columbia, Vancouver, B.C., Canada. Contact: Prof. Siegfried Schaible, Faculty of Business Administration and Commerce, University of Alberta, Edmonton, Alberta, Canada T6G 2G1; Telex 037-2979, Telephone 403-432-5027.
- August 28-30: "Bonn Workshop on Combinatorial Optimization", Institute of Operations Research, Bonn, Federal Republic of Germany. Contact: See 1982, late summer.
- September 15-17: "2nd IFAC Workshop on Control Applications of Nonlinear Programming and Optimization", Oberpfaffenhofen, Federal Republic of Germany. Contact: Dr. Klaus Weil, Institut für Dynamik der Flugsysteme, DFVLR, Oberpfaffenhofen, D-8031 Wessling, F.R.G.; telephone (0 81 53) 2 81.

*See other side. please.....*

September 16-18: "6th International Seminar on Algorithms for Production Control and Scheduling", Karlovy Vary, Czechoslovakia. Contact: Ing. Jifi Kral, House of Technology, Gorkeho nam. 23, 11282 Praha 1, Czechoslovakia.

December 10-12: "19th IEEE Conference on Decision and Control", Albuquerque, New Mexico, U.S.A. Submission deadline 31 March 1980. Contact: Prof. Michael K. Sain, Dept. Electrical Engineering, Notre Dame University, South Bend, IN 46556, U.S.A.

1981

January 5-6: "Mathematical Programming: Testing and Validating Algorithms and Software". U. S. National Bureau of Standards, Boulder, Colorado. Organized by the Committee on Algorithms of the MPS, the Bureau of Standards, and the Department of Energy. Contact: Dr. Richard H. F. Jackson, Center for Applied Mathematics, National Bureau of Standards, Washington, D.C. 20234; telephone 301-921-3855.

January 26-31: "Mathematische Optimierung", Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Federal Republic of Germany. Contact: Institut für Ökonometrie und Operations Research (see 1982, late summer).

April 6-8: "International Congress on Mathematical Programming", Rio de Janeiro, Brazil. Contacts: Professor R.W. Cottle, Dept. Operations Research, Stanford University, Stanford, CA94305, U.S.A.; Professor Milton Kelmanson, Pontificia Univesidade Catolica, Departamento de Engenharia Eletrica, Rua Marques de Sao Vicente, 225, Rio de Janeiro, R.J., Brazil; Professor B. Korte, Bonn (see 1982, late summer).

July 13-24: "NATO Advanced Research Institute on Nonlinear Optimization", Cambridge, England. Contact: Professor M.J.D. Powell, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, England. Sponsored by the MPS.

1982

Late summer: Eleventh International Symposium on Mathematical Programming in Bonn, Federal Republic of Germany. Contact: Institut für Ökonometrie und Operations Research Universität Bonn, Nassestrasse 2, 5300 Bonn 1, Federal Republic of Germany; Telex 886657 unibo b, Telephone (02221) 739285. Official triennial meeting of the MPS.

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