N°49

MARCH 1996

OPTIMA

MATHEMATICAL PROGRAMMING SOCIETY NEWSLETTER

Computational Advances Using the Reformulation-Linearization Technique (RLT) to Solve Discrete and Continuous Nonconvex Problems

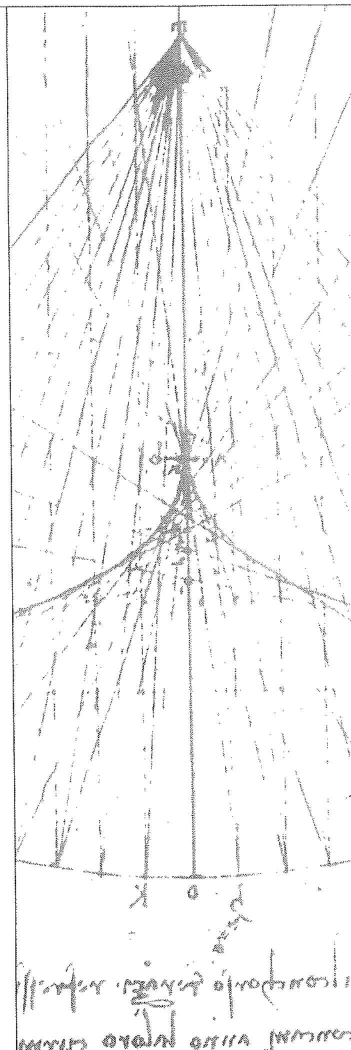
Hanif D. Sherali
Department of Industrial & Systems Engineering
Virginia Polytechnic Institute & State University
E-mail:
hanifs@vt.edu

Warren P. Adams
Department of Math Sciences
Clemson University
E-mail:
wadams@clemson.clemson.edu

Introduction. Discrete and continuous nonconvex programming problems arise in a host of practical applications in the context of production planning and control, location-allocation, distribution, economics and game theory, process design, and engineering design situations. Several recent advances have been made in the development of branch-and-cut algorithms for discrete optimization problems and in polyhedral outer-approximation methods for continuous nonconvex programming problems. At the heart of these approaches is a sequence of linear programming relaxations that drive the solution process, and the success of such algorithms is strongly tied in with the strength or tightness of these relaxations.

This article describes the application of a *Reformulation-Linearization-Technique (RLT)* that has been developed for generating such tight linear programming relaxations for not only constructing exact solution algorithms but also to design powerful heuristic procedures for large classes of discrete combinatorial and continuous nonconvex programming problems. Our work initially focused on *0-1 and mixed 0-1 linear and polynomial programs* ([6], [7], [8]), and later branched into the more general family of *continuous, nonconvex polynomial programming problems*. (An earlier survey appeared in [14].) For the family of mixed 0-1 polynomial programs in n 0-1 variables, we have ([11, 12]) developed an n-level hierarchy, with the n-th level providing an explicit algebraic characterization of the convex hull of feasible solutions. The RLT essentially consists of two steps – **a reformulation step** in which additional nonlinear valid inequalities are automatically generated and a **linearization step** in which each product term is replaced by a single continuous variable. The level of the hierarchy directly corresponds to the degree of the polynomial terms produced during the reformulation stage. Hence, in the reformulation phase, given a value of the level $d \in \{1, \ldots, n\}$, RLT constructs various polynomial factors of degree d comprised of the product of some d binary variables x_j or their complements $(1 - x_j)$. These factors are then used to multiply each of the defining constraints in the problem (including the variable bounding restrictions), to create a (nonlinear) polynomial mixed-integer zero-one programming problem. Suitable additional constraint factor products can be used to further enhance the procedure. (In general, for a variable x_j restricted to lie in the interval $[\ell_j, u_j]$, the nonnegative expressions $(x_j - \ell_j)$ and $(u_j - x_j)$ are referred to as *bound factors*, and for a structural

PAGE TWO ▶



NUMBER 49

OPTIMA

continued Computational Advances Using RLT

inequality $\alpha \cdot x \geq \beta$, for example, the expression $\alpha \cdot x \geq \beta \geq 0$ is referred to as a *constraint factor.*) After using the relationship $x_j^2 = x_j$ for each binary variable x_j, $j = 1, \ldots, n$, the linearization phase substitutes a single variable w_J and v_{Jk}, respectively, in place of each nonlinear term of the type $\pi_{j \in J} x_j$, and $y_k \pi_{j \in J} x_j$, where y represents the set of continuous variables. Hence, relaxing integrality, the nonlinear polynomial problem is linearized into a higher dimensional polyhedral set X_d defined in terms of the original variables (x, y) and the new variables (w, v). Denoting the projection of X_d onto the space of the original (x, y)-variables as X_{Pd}, it is shown that as d varies from 1 to n, we get,

$$X_{P0} \supseteq X_{P1} \supseteq X_{P2} \supseteq \ldots \supseteq X_{Pn} \equiv \text{conv}\,(X)$$

where X_{P0} is the ordinary linear programming relaxation, and conv (X) represents the convex hull of the original feasible region X.

The hierarchy of higher-dimensional representations produced in this manner markedly strengthens the usual continuous relaxation, as is evidenced not only by the fact that the convex hull representation is obtained at the highest level but that in computational studies on many classes of problems, even the first level linear programming relaxation helps design algorithms that significantly dominate existing procedures in the literature, producing tight lower bounds and often times yielding an optimal solution. Hence, this general approach holds the promise for exploiting available linear programming technology to effectively solve larger and more diffiult nonconvex problems than previously possible.

Moreover, the theoretical implications of this hierarchy are noteworthy; the resulting representations subsume and unify many published linearization methods for nonlinear 0-1 programs, and the algebraic representation available at level n promotes new methods for identifying and characterizing facets and valid linear inequalities in the original variable space as well as for providing information that directly bridges the gap between discrete and continuous sets. Indeed, since the level-n formulation characterizes the convex hull, all valid inequalities in the original variable space must be obtainable via a suitable projection; thus such a projection operation serves as an all-encompassing tool for generating valid inequalities. In this spirit, a framework for characterizing classes of facets through a sequential lifting procedure for the *Boolean quadric polytope* [26] has been devised, with new classes of facets subsuming the known clique, cut, and generalized cut inequality facets emerging. In addition, new classes of facets have been characterized for the *GUB knapsack polytope* through a polynomial-time sequential-simultaneous lifting procedure [24]. Known lower bounds on the coefficients of lifted facets derived from minimal covers associated with the ordinary knapsack polytope have been tightened. For the *set partitioning polytope* [25], a number of published valid inequalities along with constraint tightening procedures have been shown to be automatically captured within the first- and second-level relaxations themselves. A variety of partial applications of the RLT scheme have also been developed in order to delete fractional linear programming solutions while tightening the relaxation in the vicinity of such solutions.

The hierarchy of relaxations emerging from the RLT can be intuitively viewed as "stepping stones" between continuous and discrete sets, leading from the usual linear programming relaxation to the convex hull at level-n. By inductively progressing along these stepping-stone formulations, we have studied some novel *persistency* issues for certain constrained and unconstrained pseudo-Boolean programming problems. Given the tight linear programming relaxations afforded by RLT, a pertinent question that can be raised is that if we solve a particular d^{th} level representation in the RLT hierarchy, and some of the n variables turn out to be binary valued at optimality to the underlying linear program, then can we expect these binary values to "*persist*" at optimality to the original problem? In [5], we derive sufficient conditions in terms of the dual solution that guarantee such a persistency result. For the unconstrained pseudo-Boolean program, we show that for $d = 1$ or for $d \geq n - 2$, persistency always holds. However, using an example with $d = 2$ and $n = 5$, we show that without the additional prescribed sufficient conditions, persistency will not hold in general. These results are also extended to constrained polynomial 0-1 programming problems. In particular, the analysis here reveals a class of 0-1 linear programs that possess the foregoing persistency property. Included within this class as a special case is the popular vertex packing problem, shown earlier in the literature to possess this property.

Recently [15], we have extended our RLT framework to generate a new hierarchy of relaxations leading to the convex hull representation based on the development of more generalized product factors, other than simply x_j and $(1 - x_j)$, for $j = 1, \ldots, n$, in the reformulation phase. In addition, this hierarchy embeds within its construction stronger logical implications than only $x_j^2 = x_j \; \forall j = 1, \ldots, n$. As a result, it not only subsumes the previous development but also provides the opportunity to exploit frequently-arising special structures such as generalized/variable upper bounds, covering, partitioning, and packing constraints, as well as sparsity.

Although the Reformulation-Linearization Technique was originally designed to employ factors involving *zero-one* variables in order to generate zero-one (mixed-integer) polynomial programming problems that are subsequently re-linearized, the approach has also been extended to solve continuous, bounded variable polynomial programming problems. Problems of this type involve the optimization of a polynomial objective function subject to polynomial constraints in a set of continuous, bounded variables, and arise in numerous applications in engineering design, production, location, and distribution problems.

In [29] we prescribe an RLT process that employs suitable polynomial factors to generate additional polynomial constraints through a multiplication process which, upon linearization through variable redefinitions, produces a linear programming relaxation. The resulting relaxation is used in concert with a suitably designed partitioning technique to develop an algorithm that is proven to converge to a global optimum for this problem. While RLT essentially operates on polynomial functions having integral exponents, many engineering design applications lead to polynomial programs having general rational exponents. For such problems, we have recently developed a global optimization technique [10] by introducing a new level of approximation at the reformulation step and, accordingly, redesigning the partitioning scheme in order to induce the overall sequence of relaxations generated to become exact in the limit. Our ongoing investigations include the extension of the RLT theory to accommodate discrete-valued variables in general, as opposed to the more restrictive 0-1 case. We have already verified that an analogous hierarchy results, once again leading to the convex hull representation at level n, with a paper forthcoming. (This was presented at the *INFORMS* Meeting, New Orleans, Fall, 1995.)

The insights obtained in the development of the RLT have also led to a variety of related linearization results. For example, the specially-structured families of upper bounding functions for quadratic pseudo-Boolean functions known as "paved upper planes" and "roofs" were explained in terms of a Lagrangean dual to the level-one linearization, with the bound called the "height" being the optimal objective function value to this formulation [2]. An offshoot of this study allowed for the extension of the published persistency results on "roof duality" to polynomial pseudo-Boolean functions [1]. In fact, the persistency results on the hierarchical levels led to the development of an entire new linearization strategy which, while not producing convex hull representations, characterize an entire family of persistent linear reformulations of various constrained and unconstrained 0-1 polynomial programs [9], encompassing and generalizing all known persistent formulations.

Over the remainder of this article, we now focus on some specialized RLT designs that have been used to solve various specific discrete and continuous nonconvex programming problems, and relate our computational experience obtained in these instances.

Zero-one quadratic programs and the quadratic assignment problem. The zero-one quadratic programming problem seeks to minimize a general quadratic function in n 0-1 variables, subject to linear equality and/or inequality constraints. In [6] we presented a new linearization technique that, in effect, evolved to become precisely the level-one relaxation of the RLT hierarchy discussed above. This relaxation was shown to theoretically dominate other existing linearizations and was shown to computationally produce far tighter lower bounds. In these computations, we solved quadratic set covering problems having up to 70 variables and 40 constraints. For example, for this largest size problem, where the optimum objective value was 1312, our relaxation produced an initial lower bound of 1289 at the root node, and enumerated 14 nodes to solve the problem in 79 cpu seconds on an IBM 3081 Series D24 Group K computer. When the same algorithmic strategies were used on a relaxation that did not include the special RLT constraints, the initial lower bound obtained was 398, and the algorithm enumerated 2130 nodes, consuming 197 cpu seconds.

The first and second level RLT relaxations have also been used to develop strong lower bounds for the quadratic assignment problem. Because the assignment constraints are equality restrictions, these RLT relaxations are produced by simply multiplying the constraints with individual or with pairs of variables, respectively, as pointed out in Section 7 of [12]. In [3], we show that the lower bound produced by the first level relaxation itself subsumes a multitude of known lower bounding techniques in the literature, including a host of matrix reduction strategies. By designing a heuristic dual ascent procedure for the level-one relaxation and by incorporating dual-based cutting planes within an enumerative algorithm, an exact solution technique [4] has been developed and tested that can competitively solve problems up to size 17. In an effort to make this algorithm generally applicable, no special exploitation of flow and/or distance symmetries was considered. As far as the strength of the RLT relaxation is concerned, on a set of standard test problems of sizes 8-20, the lower bounds produced by the dual ascent procedure uniformly dominated 12 other competing lower bounding schemes except for one problem of size 20, where our procedure yielded a lower bound of 2142, while an eigenvalue-based procedure produced a lower bound of 2229, the optimum value being 2570 for this problem. Recently, Resende *et al.* (*Operations Research*, 1995) have been able to solve the first level RLT relaxation exactly for problems of size up to 30 using an interior-point method that employs a preconditioned conjugate gradient technique to solve the system of equations for computing the search directions. (For the aforementioned problem of size 20, the exact solution value of the lower bounding RLT relaxation turned out to be 2182, compared to our dual ascent value of 2142.) As a point of interest, we mention that Ramachandran and Pekny (*INFORMS*, Fall 1995) have been conducting research on precisely the second and higher-level RLT relaxation for this problem, promoting encouraging preliminary results.

We have also applied RLT to the problem of assigning aircraft to gates at an airport, with the objective of minimizing passenger walking distances [18]. The problem is modeled as a variant of the quadratic assignment problem with partial assignment and set packing constraints. The quadratic problem is then equivalently linearized by applying the first-level of the RLT. In addition to simply linearizing the problem, the application of this technique generates additional constraints that provide a tighter linear programming representation. Since even the first-level relaxation can get quite large, we investigate several alternative relaxations that either delete or aggregate classes of RLT constraints. All these relaxations are embedded in a heuristic that solves a sequence of such relaxations, automatically selecting at each stage the tightest relaxation that can be solved with an acceptable estimated effort, and based on the solution obtained, it fixes a suitable subset of variables to 0-1 values. This process is repeated until a feasible solution is constructed. The procedure was computationally tested using realistic data obtained from *USAir* for problems having up to 7 gates and 36 flights. For all the test problems ranging from 4 gates and 36 flights to 7 gates and 14 flights, for which the size of the first-level relaxation was manageable (having 14, 494 and 4,084 constraints, respectively, for these two problem sizes), this initial relaxation itself always produced an optimal 0-1 solution.

Continuous and discrete bilinear programming problems. The well-known nonconvex, NP-hard bilinear programming problem seeks to *Minimize* $\phi(x, y) \equiv c^t x + d^t y + x^t G y$, *subject to* $(x, y) \in Z \cap \Omega$, where $x \in R^n$, $y \in R^m$, Z is a polyhedron in R^{n+m}, and Ω is a hyperrectangle, representing finite lower and upper bounds on the variables. The problem considered is *separably constrained* if Z is separable over x and y, as is often assumed to be the case, and is *jointly constrained*, otherwise. (The latter class of problems are more difficult to solve in practice.) Problems of this type find numerous applications in economics and game theory, location theory, dynamic assignment and production problems, and various risk management problems.

An enhanced first-level RLT relaxation is designed in [17] for such problems by using all pairwise products of structural and bounding constraint factors. For special classes of polytopes, this is shown to yield an exact convex hull representation. (Also, see [16].) More generally, this yields a linear programming relaxation that is embedded within a provably convergent branch-and-bound algorithm. Computational experiments were conducted on problems of size up to 14 variables, including both separably and jointly constrained test problems from the literature, as well as on randomly generated problems. For all 15 instances of separably constrained problems and a great majority of jointly constrained problems (15 out of 18 instances), the initial linear programming relaxation itself

continued Computational Advances Using RLT

solved the underlying bilinear problem. Whenever this was not the case, the initial gap between the lower and upper bounds was close enough to produce an optimum after enumerating only a few nodes (fewer than 11). This performance exhibits a significant improvement over the previously best algorithm based on convex envelopes, which consumed far more effort and was unable to solve several of the test problems within the set computational limits.

In [8], we consider a variation of the bilinear programming problem in which one of the sets of variables is restricted to be binary valued, representing discrete location or investment decisions, and where the continuous and the binary variables are separably constrained. A partial, modified, first-level RLT relaxation is constructed in which bound factors based on one set of variables are appropriately used to multiply constraints involving the other set of variables, followed by a linearization of the cross-product terms. The proposed algorithm additionally employs Benders' cuts, disjunctive cuts, and Lagrangian relaxation strategies. Again, very favorable computational results have been reported on an extensive set of test problems. In particular, problems having up to 100 continuous variables and 70 binary variables were solved to optimality within about 250 cpu seconds on an IBM 3081 computer. For three different classes of problems differing in signs on the objective coefficients and the density of the constraints in the 0-1 variables, the average initial lower bounds over 6 instances of various sizes were, respectively 99.62%, 99.24%, and 83%. Hence, evidently, the success of the algorithm is strongly related to the tightness of the bounds produced by RLT.

In [23], we have applied a similar RLT approach to solve general linear complementarity problems (LCP) where the underlying matrix *M* does not possess any special property. (Also, see [22].) Formulating such problems equivalently as mixed-integer bilinear programming problems, the RLT process described above for the latter class of problems was enhanced by incorporating constraint factor cross-products as well, and by exploiting the fact that the optimal objective function value is zero if and only if an LCP solution exists. On a total of 70 test problems using negative definite and indefinite matrices *M* of size up to 25 x 25, all problem instances except for one were solved at the root node itself via the solution of a single linear program. For the one exception, the LP solver CPLEX quit after hitting a limit of 10,000 iterations. However, when a subgradient based Lagrangian dual approach was applied to this problem, the LCP was again solved at the root node itself. In general, although the Lagrangian dual approach was unable to attain the same tight bounds as CPLEX did due to convergence difficulties, and as a result, it sometimes led to an enumeration of 2 or 3 nodes, it was still 3-4 times faster in terms of the overall effort required as compared with the CPLEX based approach.

Continuous and discrete location-allocation problems. The RLT strategy has been used to derive very effective algorithms for capacitated, multifacility, location-allocation problems that find applications in service facility or warehouse location, or manufacturing facility flow-shop design problems. Given n demand locations (customers or machines) having known respective demands, the problem is to simultaneously determine the *locations* of some m supply centers (service facilities, warehouses, interacting machines, or tooling centers) having known respective capacities, and an *allocation* of products from each source to each destination, in order to minimize total distribution costs. For the *rectilinear distance* variant of this problem that arises in applications where the flow of goods or materials occurs along grids of city streets or factory aisles, the cost is directly proportional to the shipment volume and the rectilinear distance through which this shipment occurs. This problem can be equivalently reformulated as a mixed-integer, zero-one, bilinear programming problem of the general form studied in [8]. We specialized the discussed level-one RLT procedure for the above problem in [27] and were able to solve these difficult nonconvex problems having up to 5 sources and 20 customer locations to optimality. In addition, because of the tight relaxations obtained, this algorithm also provides an efficient heuristic which upon premature termination is capable of obtaining provably good quality solutions (within 5-10% of optimality) for larger sized problems.

Another interesting location-allocation problem arises in the case when the per unit transportation cost penalty is proportional to the squared Euclidean distance between the supply and destination points. In contrast with the transformation used for the rectilinear distance problem that essentially analyzes that problem over the location decision space, we [30] projected the squared Euclidean distance problem onto the space of the allocation-variables alone, transforming it into one of minimizing a concave quadratic function over the transportation constraints. For this equivalent representation, we devised a suitable application of the RLT concept by generating a selective set of first level bound factor based RLT constraints. Our computational tests have revealed that the bounds obtained from this relaxation are substantially superior to four different lower bounds obtained using standard techniques. Computational experience reveals that the initial linear program itself produces solutions within 2-4% of optimality and that this procedure significantly enhances the size of problems solvable by a branch-and-bound algorithm. We have solved problems having (m,n)= (6,120) – (20,60) within about 150 cpu secs on an IBM 3090 computer, while the methods employing four standard lower bounding techniques (previously developed by others) were able to handle problems of size up to only $m = 4$ and $n = 6$ within 370 cpu secs on the same computer.

Predating this work, we had also studied a discrete variant of the location-allocation problem in which the m capacitated service facilities are to be assigned in a one-to-one fashion to some m discrete sites in order to serve the n customers, where the cost per unit flow is determined by some general facility-customer separation based penalty function [13]. This problem also turns out to have the structure of a separably constrained mixed-integer bilinear programming problem, and a partial first level RLT relaxation that includes only a subset of the constraints developed in [8], some in an aggregated form, was used to generate lower bounds. A set of 16 problems with (m,n) ranging up to (7,50)-(11,11) were solved using a Benders' partitioning approach. For these problem instances, even the partial, aggregated first level RLT relaxation produced lower bounds within 90-95% of optimality.

Indefinite quadratic programs and polynomial programming problems. In [31], we have developed a global optimization procedure for linearly constrained indefinite/concave-minimization quadratic programming problems. These are hard nonconvex programming problems that can have many local optima that differ significantly from the global optimal solution. We have designed and tested RLT based relaxations for such problems which depart somewhat from previous approaches in that the relaxations are not purely linear, but they retain a critical, manageable degree of nonlinearity in terms of separable quadratic functions. Specifically, a selected set of first level bound factor based as well as constraint factor based RLT constraints are generated and are augmented by convex, variable upper bounding restrictions of the type $x_k^2 \leq w_{kk}$ for each variable x_k in the problem, where w_{kk} is the linearized variable representing the nonlinear term x_k^2. Furthermore, various range-reduction strategies were devised based on the eigenstructure of the problem, logical tests on the constraints, and the Lagrangian dual based lower bound produced via the RLT relaxation. These strategies were all embedded in a branch-and-bound framework to provide global optimal solutions to the underlying nonconvex problems. The tightness of the RLT relaxations was evident from the computational results presented on solving several test problems from the literature of size up to 20 variables. For many instances, the problem was solved at the initial node itself, while for others, only a few nodes were enumerated in solving the problem. Among the former instances of problems was one particularly notorious jointly constrained bilinear program that had required 11 nodes to be enumerated by our previous RLT based algorithm for this class of problems and over 105 nodes for a competing convex envelope based approach. Also, one published test problem in 20 variables had not previously been solved to optimality. In addition, we attempted to solve a set of larger (up to 50 variables) randomly generated problems obtained using an available test problem generator. In all the seven instances solved, the lower bound obtained at node zero was within 95.2-99.9% of optimality. Hence, this technique affords a tool to derive near optimal solutions to hard nonconvex problems via a single (essentially linear) relaxation.

We have also devised similar RLT based strategies to solve polynomial programming problems. A specialization to the water distribution network design problem is described in [28] where global optimal solutions to a standard test problem from the literature, and some of its variants, are presented for the first time ever. (A recent paper by G. Eiger, U. Shamir and A. Ben-Tal in *Water Resources Research*, 30(8), 2637-2646, provides the only other global optimization approach for this problem, and also solves certain other instances to optimality.) For general polynomial programming problems, we present in [32] some theoretical and computational dominance results of applying the RLT lower bounding scheme directly as in [29] to the original polynomial program, versus applying it to an equivalent quadratic polynomial program that is obtained through a standard successive transformation process. This dominance holds even if all alternative ways of making such a transformation are *simultaneously* included within the quadratic reformulation.

In [33], we present various new classes of RLT constraints that can be used to tighten relaxations. For univariate polynomial programs, these constraints use certain special squared grid factor products based on a discretization of the bounding interval in one case and certain squared Lagrange interpolation constraints based also on a discretization in the second case. In each case, the stated constraints define valid inequalities derived by suitably composing products of factors, where each factor is based on a grid point rather than simply on a bounding interval end-point. For example, on a sample of problems having degree 3, 4, and 6 with respective optimal values –4.5, 0, and 7, respectively, our RLT procedure produced the optimum via the initial lower bounding linear program itself, while a standard exponential transformation based technique produced initial lower bounds of –1311, –22354, and –5519681, respectively. Similarly, we solved some standard constrained multivariate engineering design test problems from the literature having n variables, and polynomial objective and constraint functions of degree δ. Again, the tightness of the lower bounds produced by the RLT linear programming relaxation at the root node was instrumental in algorithmic efficiency. For example, a set of problems having $(n, \delta) = (2, 4), (4, 4), (10, 3)$, and $(13, 2)$ with respective optimal values of –16.7389, 17.01417, –1768.807, and 97.588, respectively, the initial lower bounds produced by the RLT relaxation were –16.7389, 16.75834, –1795.572, and 97.588, respectively. Hence, only a few nodes (often just 1-3) were required to solve several of these problems to 1% of optimality.

Conclusions and extensions. In this article, we have described our experience in designing specialized RLT based relaxations for solving various specific classes of discrete and continuous nonconvex problems. We are currently investigating RLT designs for many other applications, notably, some telecommunication design problems as well as the development of general purpose algorithmic strategies for solving linear mixed-integer 0-1 programming problems and continuous polynomial programming problems. Specific special structures inherent within such problems can be exploited by the RLT process as discussed in [15].

In conclusion, there is one particular helpful comment that potential users of RLT need to bear in mind. As evident from our discussion, in using the RLT approach, one has to contend with the (repeated) solution of large-scale linear programming problems that are generated to provide tight polyhedral outer-approximations for the convex hull of feasible solutions. By the nature of the RLT process, these linear programs possess a special structure induced by the replicated products of the original problem constraints (or its subset) with certain designated variables. At the same time, this process injects a high level of degeneracy in the problem since blocks of constraints automatically become active whenever the factor expression that generated them turns out to be zero at any feasible solution. As a result, simplex-based procedures and even interior-point methods experience difficulty in coping with such reformulated linear programs (see [8] for some related computational experience). On the other hand, a Lagrangian duality based scheme can not only exploit the inherent special structures but can quickly provide near optimal primal and dual solutions that serve the purpose of obtaining tight lower and upper bounds. References [19, 20, 21, 34] provide a discussion on related literature, as well as new subgradient deflection and stepsize techniques and procedures, for generating both primal and dual solutions via such a Lagrangian dual based approach.

Acknowledgement. This material is based upon research supported by the National Science Foundation under Grant Number DMI-9521398.

References Related to Authors' RLT Research

[1] W. P. Adams, A. Billionnet, and A. Sutter, "Unconstrained 0-1 Optimization and Lagrangean Relaxation," *Discrete Applied Mathematics*, Vol. 29, Nos. 2-3, pp. 131-142, 1990.

[2] W. P. Adams and P. M. Dearing, "On the Equivalence Between Roof Duality and Lagranian Duality for Unconstrained 0-1 Quadratic Programming Problems," *Discrete Applied Mathematics*, Vol. 48, No. 1, pp. 1-20, 1994.

continued Computational Advances Using RLT

[3] W. P. Adams and T. A. Johnson, "Improved Linear Programming-Based Lower Bounds for the Quadratic Assignment Problem," *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, "Quadratic Assignment and Related Problems," eds. P. M. Pardalos and H. Wolkowicz, 16, pp. 43-75, 1994.

[4] W. P. Adams and T. A. Johnson, "An Exact Solution Strategy for the Quadratic Assignment Problem Using RLT-Based Bounds," Working paper, Department of Mathematical Sciences, Clemson University, Clemson, SC, 1996.

[5] W. P. Adams, J. B. Lassiter and H. D. Sherali, "Persistency in 0-1 Optimization," under revision for *Mathematics of Operations Research* (Manuscript, 1993).

[6] W. P. Adams and H. D. Sherali, "A Tight Linearization and an Algorithm for Zero-One Quadratic Programming Problems," *Management Science*, 32(10), pp. 1274-1290, 1986.

[7] W. P. Adams and H. D. Sherali, "Linearization Strategies for a Class of Zero-One Mixed Integer Programming Problems," *Operations Research*, 38(2), pp. 217-226. 1990.

[8] W. P. Adams and H. D. Sherali, "Mixed-Integer Bilinear Programming Problems," *Mathematical Programming*, 59(3), pp. 279-306, 1993.

[9] J. B. Lassiter and W. P. Adams, "Strategies for Converting Various 0-1 Polynomial Programs into Persistent Mixed 0-1 Linear Problems," Working paper, Department of Mathematical Sciences, Clemson University, Clemson, SC, 1994.

[10] H. D. Sherali, "Global Optimization of Nonconvex Polynomial Programming Problems Having Rational Exponents," Working paper, Department of Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, 1996.

[11] H. D. Sherali and W. P. Adams, "A Hierarchy of Relaxations Between the Continuous and Convex Hull Representations for Zero-One Programming Problems," *SIAM Journal on Discrete Mathematics*, 3(3), pp. 411-430, 1990.

[12] H. D. Sherali and W. P. Adams, "A Hierarchy of Relaxations and Convex Hull Characterizations for Mixed-Integer Zero-One Programming Problems," *Discrete Applied Mathematics*, 52, pp. 83-106, 1994.

[13] H. D. Sherali and W. P. Adams, "A Decomposition Algorithm for a Discrete Location-Allocation Problem," *Operations Research*, pp. 878-900, 1984.

[14] H. D. Sherali and W. P. Adams, "A Reformulation-Linearization Technique (RLT) for Solving Discrete and Continuous Nonconvex Programming Problems," Vol. XII-A, pp. 61-78, *Mathematics Today*, special issue on *Recent Advances in Mathematical Programming*, ed. O. Gupta, 1994.

[15] H. D. Sherali, W. P. Adams, and P. Driscoll, "Exploiting Special Structures in Constructing a Hierarchy of Relaxations for 0-1 Mixed Integer Problems," Working paper, Department of Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0118, 1994.

[16] H. D. Sherali and A. Alameddine, "An Explicit Characterization of the Convex Envelope of a Bivariate Bilinear Function Over Special Polytopes," *Annals of Operations Research, Computational Methods in Global Optimization*, eds. J. B. Rosen and P. Pardalos, 25(1-4), pp. 197-214, 1990.

[17] H. D. Sherali and A. Alameddine, "A New Reformulation-Linearization Technique for Solving Bilinear Programming Problems," *Journal of Global Optimization*, 2, pp. 379-410, 1992.

[18] H. D. Sherali and E. L. Brown, "A Quadratic Partial Assignment and Packing Model and Algorithm for the Airline Gate Assignment Problem," *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, "Quadratic Assignment and Related Problems," eds. P. M. Pardalos and H. Wolkowicz, 16, pp. 343-364, 1994.

[19] H. D. Sherali, G. Choi, and Z. Ansari, "Memoryless and Limited Memory Space Dilation and Reduction Algorithm," under revision for the *Journal of Optimization Theory and Applications*.

[20] H. D. Sherali, G. Choi, and C. H. Tuncbilek, "A Variable Target Value Method," under revision for *Mathematical Programming*.

[21] H. D. Sherali and G. Choi, "Recovery of Primal Solutions When Using Subgradient Optimization Methods to Solve Lagrangian Duals of Linear Programs, *Operations Research Letters*, to appear.

[22] H. D. Sherali, R. S. Krishnamurthy and F. A. Al-Khayyal, "An Enhanced Intersection Cutting Plane Approach for Linear Complementarity Problems," *Journal of Optimization Theory and Applications*, to appear.

[23] H. D. Sherali, R. Krishnamurthy and F. A. Al-Khayyal, "A Reformulation-Linearization Approach for the General Linear Complementarity Problem," Presented at the Joint National ORSA/TIMS Meeting, Phoenix, Arizona, October 31-November 3, 1993.

[24] H. D. Sherali and Y. Lee, "Sequential and Simultaneous Liftings of Minimal Cover Inequalities for GUB Constrained Knapsack Polytopes," *SIAM Journal on Discrete Mathematics*, 8(1), pp. 133-153, 1995.

[25] H. D. Sherali and Y. Lee, "Tighter Representations for Set Partitioning Problems Via a Reformulation-Linearization Approach," *Discrete Applied Mathematics*, to appear.

[26] H. D. Sherali, Y. Lee and W. P. Adams, "A Simultaneous Lifting Strategy for Identifying New Classes of Facets for the Boolean Quadric Polytope," *Operations Research Letters*, 17(1), pp. 19-26, 1995.

[27] H. D. Sherali, S. Ramachandran and S. Kim, "A Localization and Reformulation Discrete Programming Approach for the Rectilinear Distance Location-Allocation Problem, "*Discrete Applied Mathematics*, 49(1-3), pp. 357-378, 1994.

[28] H. D. Sherali and E. P. Smith, " A Global Optimization Approach to a Water Distribution Network Problem," under revision for the *Journal of Global Optimization*.

[29] H. D. Sherali and C. H. Tuncbilek, "A Global Optimization Algorithm for Polynomial Programming Problems Using a Reformulation-Linearization Technique," *Journal of Global Optimization*, 2, pp. 101-112, 1992.

[30] H. D. Sherali and C. H. Tuncbilek, "A Squared-Euclidean Distance Location-Allocation Problem," *Naval Research Logistics*, 39, 447-469, 1992.

[31] H. D. Sherali and C. H. Tuncbilek, "A Reformulation-Convexification Approach for Solving Nonconvex Quadratic Programming Problems," *Journal of Global Optimization*, 7, pp. 1-31, 1995.

[32] H. D. Sherali and C. H. Tuncbilek, "Comparison of Two Reformulation-Linearization Technique Based Linear Programming Relaxations for Polynomial Programming Problems," Working paper, Department of Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1995.

[33] H. D. Sherali and C. H. Tuncbilek, "New Reformulation-Linearization Technique Based Relaxations for Univariate and Multivariate Polynomial Programming Problems," Working paper, Department of Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1996.

[34] H. D. Sherali and O. Ulular, "A Primal-Dual Conjugate Subgradient Algorithm for Specially Structured Linear and Convex Programming Problems," *Applied Mathematics and Optimization*, 20(S-15), pp. 193-221, 1989.

OPTIMA

JOURNALS

Volume 71, No. 2

E.H. Aghezzaf, T.L. Magnanti and L.A. Wolsey, *"Optimizing constrained subtrees of trees."*

C. Blair, *"A closed-form representation of mixed-integer program value functions."*

S.E. Karisch and F. Rendl, *"Lower bounds for the quadratic assignment problem via triangle decompositions."*

A.V. Goldberg and R. Kennedy, *"An efficient cost scaling algorithm for the assignment problem."*

J.V. Burke and M.C. Ferris, *"A Gauss-Newton method for convex composite optimization."*

R. Sridhar, *"Superfluous matrices in linear complementarity."*

U. Brännlund, *"A generalized subgradient method with relaxation step."*

E.D. Andersen and K.D. Andersen, *"Presolving in linear programming."*

Volume 71, No. 3

M. Conforti and G. Cornuéjols, *"Balanced 0,±1-matrices, bicoloring and total dual integrality."*

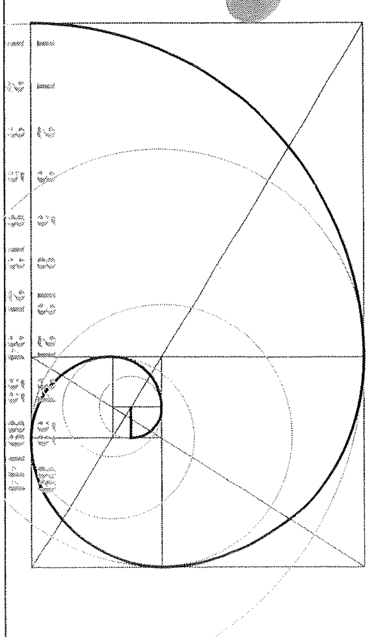
S. Filipowski, *"On the complexity of solving feasible systems of linear inequalities specified with approximate data."*

B. De Schutter and B. De Moor, *"The extended linear complementarity problem."*

T.L. Magnanti and G. Perakis, *"A unifying geometric solution framework and complexity analysis for variational inequalities."*

F.B. Shepherd, *"Applying Lehman's theorems to packing problems."*

G. Pritchard, G. Gürkan and A.Y. Özge, *"A note on locally Lipschitzian functions (Short Communication)."*



Nominations for the A.W. Tucker Prize Are Invited

The Mathematical Programming Society invites nominations for the A.W. Tucker Prize for an outstanding paper authored by a student. The award will be presented at the International Symposium on Mathematical Programming in Lausanne (24-29 August 1997). All students, graduate and undergraduate, are eligible. Nominations of students who have not yet received the first university degree are especially welcome. In advance of the Symposium an award committee will screen the nominations and select at most three finalists. The finalists will be invited, but not required, to give oral presentations at a special session of the Symposium. The award committee will select the winner and present the award prior to the conclusion of the Symposium. The members of the committee for the 1997 A.W. Tucker Prize are: Kurt Anstreicher, Rolf. H. Mohring, Jorge Nocedal, J.-P. Vial (Chairman) and David Williamson.

Eligibility The paper may concern any aspect of mathematical programming; it may be original research, an exposition or survey, a report on computer routines and computing experiments, or a presentation of a new and interesting application. The paper must be solely authored and completed after January 1994. The paper and the work on which it is based should have been undertaken and completed in conjunction with a degree program.

Nominations must be made in writing to the chairman of the award committee by a faculty member at the institution where the nominee was studying for a degree when the paper was completed. Letters of nomination must be accompanied by five copies each of: the student's paper; a separate summary of the paper's contributions, written by the nominee, and no more than two pages in length; and a brief biographical sketch of the nominee.

Deadline Nominations must be sent to the chairman, as follows, and postmarked no later than December 31, 1996:

Jean-Philippe Vial
HEC/Management Studies
University of Geneva
102, Bd Carl Vogt
CH-1211 Geneva 4
Switzerland

Laboratoire Approximation & Optimisation

Université Paul Sabatiere
Toulouse, France

MODE

(Mathématiques de l'Optimisation et de la Décision) is a permanent group inside SMAI (Société de Mathématiques Appliquées et Industrielles, France) gathering people interested in: optimization (mathematical programming, variational problems, operations research) and mathematics of the decision sciences (mathematical economics, mathematics in social sciences).

Its current board committee is composed of:

J.-B. Hiriart-Urruty (President)
B. Monjardet (Vice-President)
M. Théra (Secretary)
C. Lemaréchal (Treasurer).

The report on activities of 1994 can be found in MATAPLI 43 (the newsletter of SMAI) of July 1995.

For more information please contact:

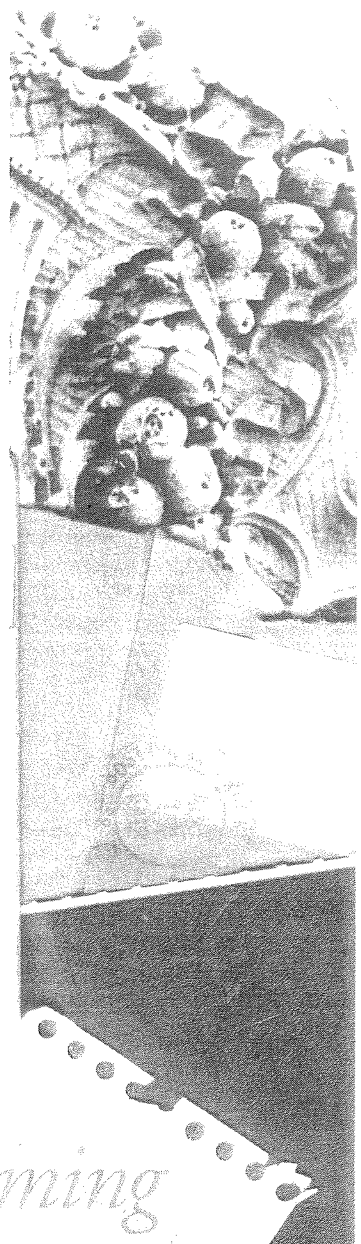
Address: 118, route de Narbonne, 31062 TOULOUSE Cedex - France.
Secrétariat: Bâtiment 1R2, Porte 25, rez de chaussée.
Telephone: 61 55 67 78
Telecopie: 61 55 61 83
E-mail: lao@cict-fr

-J.-B. HIRIART-URRUTY

O P T I M A

Conference Notes

- 5th SIAM Conference on Optimization, British Columbia May 20-22, 1996.
- 18th Symposium on Mathematical Programming with Data Perturbations George Washington University 23-24 May 1996
- IPCO V, Vancouver, British Columbia, Canada June 3-5, 1996
- Fifth International Symposium on Generalized Convexity Luminy-Marseille, France June 17-21, 1996
- 7th Stockholm Optimization Days Stockholm, Sweden June 24-25, 1996
- IFORS 96 14th Triennial Conference, Vancouver British Columbia, Canada July 8-12, 1996
- IRREGULAR 96 Santa Barbara, California Aug. 19-23, 1996
- International Conference on Nonlinear Programming Beijing, China Sept. 2-5, 1996
- Symposium on Operations Research (SOR96) Technical University Braunschweig, Germany Sept. 4-6, 1996
- Second International Symposium on Operations Research and its Applications (ISORA '96) Guilin, China Dec. 11-13, 1996
- XVI International Symposium on Mathematical Programming, Lausanne Switzerland, Aug. 1997



forthcoming **conferences**

call for PAPERS

7th Stockholm Optimization Days
Stockholm, Sweden, June 24-25, 1996

We welcome theoretical, computational and applied papers for the 7th Stockholm Optimization Days, a two-day conference on optimization, to be held at KTH (Royal Institute of Technology) in Stockholm, Sweden, June 24-25, 1996.

There will be sessions on various aspects of optimization, including nonsmooth optimization, linear and nonlinear programming, as well as applications of optimization in areas such as structural optimization and transportation.

We anticipate some 30 talks in total, out of which approximately 15 are invited presentations.

Abstracts (maximum 200 words) should be sent by May 1 (preferably by e-mail) to:
optdays@math.kth.se

or by mail to:
Optimization Days
Division of Optimization and Systems Theory
KTH
S-100 44 Stockholm
Sweden
Fax: +46 8 - 22 53 20.

Further information can be obtained from the same addresses.

The conference is financially supported by the Goran Gustafsson Foundation and the Swedish National Board for Industrial and Technical Development (NUTEK). The organizing committee consists of Ulf Brannlund, Anders Forsgren and Krister Svanberg (head), from the Division of Optimization and Systems Theory, Department of Mathematics, Royal Institute of Technology (KTH).

-KRISTER SVANBERG

call for PAPERS

SOR96 Announcement & Call for Papers

Symposium on Operations Research (SOR96)
Annual Conference of the DGOR and GMÖOR with the participation of WG 7.4 of the IFIP
Technical University
Braunschweig, Germany
September 4-6, 1996

	Section	*Chairperson*
1.	Linear Programming	Bixby, Houston
2.	Nonlinear Programming	Scholtes, Karlsruhe
3.	Combinatorial and Discrete Optimization	Burkard, Graz
4.	Graph Algorithms and Complexity	Möhring, Berlin
5.	Stochastic Models and Optimization	Mosler, Köln
6.	Scheduling	Drexl, Kiel
7.	Production	Tempelmeier, Köln
8.	Transportation	Domschke, Darmstadt
9.	Macroeconomics, Economic Theory, Games	Eichhorn, Karlsruhe
10.	Statistics and Econometrics	Kreiß, Braunschweig
11.	Marketing and Data Analysis	Gaul, Karlsruhe
12.	Information and Decision Support Systems	Derigs, Köln
13.	Banking, Finance, Insurance	Minnemann, Düsseldorf
14.	Environment, Energy, Health	Stepan, Wien
15.	Neural Networks and Fuzzy Systems	Werners, Bochum
16.	Control Theory	Hartl, Wien
17.	Simulation	Chamoni, Duisburg
18.	Practical OR (Application Reports)	Schuster, Jesteburg

Conference languages: English and German

Deadlines:

Apr 1, 1996	Sending a paper copy of the abstract
Apr 15, 1996	Sending the abstract by e-mail (preferred)
May 15, 1996	Regular registration deadline
Jul 15, 1996	Sendout of the preliminary program
Aug 1, 1996	Requests for cancellation refund
Aug 1, 1996	Receipt of a paper for the proceedings volume

Distribution and gathering of information for SOR96 will to a large extent be based on e-mail and electronic networks. So, whenever possible, use e-mail and the Web for communication. The e-mail address for all these contacts is:

sor96@tu-bs.de

Certain keywords on the subject line will trigger automatic responses, such as

Subject	*Reply*
info	returns the detailed second announcement
help	returns a description of the e-mail interface
help registration	returns a registration form
help abstract	returns an abstract form
help hotel	returns a hotel form
help manuscript	returns information concerning manuscripts for the proceedings

To communicate via the Web, start at URL

http://moa.math.nat.tu-bs.de/sor96

and follow the respective links (WWW-forms).

Organizing committee:

Prof.Dr. R. Schaßberger
Prof.Dr. S. Voß
Prof.Dr. G. Wäscher
Prof.Dr. U. Zimmermann

Program committee:

Prof. Dr. U. Derigs
Prof. Dr. W. Gaul
Prof. Dr. R. H. Möhring
K.-P. Schuster

Mailing address:
Prof. Dr. U. Zimmermann
Abt. Mathem. Optimierung
TU Braunschweig
D-38106 Braunschweig, Germany
+49(0)531-391-7550
Fax: +49(0)531-391-7559
sor96@tu-bs.de

-MICHAEL BUSSIECK
bussieck@moa.math.nat.tu-bs.de

Annals of Operations Research

Special Issue on Parallel Optimization

Parallel computing emerges as one of the most powerful and versatile tools available to address complex and large scale problems. The applications are varied, numerous and meaningful; one can mention the fields of finance, real time reaction and decision making, intelligent transportation systems, artificial intelligence, biology and chemistry, etc. Yet, the potential gains of parallel computation do not materialize easily. In fact, parallel computation challenges researchers to rethink their models and algorithms, besides imposing a few specific issues of its own (e.g., efficient data structures, performance measures, etc.). There is now a significant body of scientists in operations research and mathematical programming actively involved in addressing these issues and developing sound and efficient **Parallel Optimization** models and algorithms in a wide variety of application contexts. We intend this special issue of the *Annals of Operations Research* to capture the essence of today's state-of-the-art research in this dynamic and exciting field.

We seek original, high quality contributions that may belong, but are not restricted, to one of the following categories:

- Methodological work on models and algorithms in continuous linear and nonlinear optimization, integer and mixed integer programming, combinatorial optimization, network flows, metaheuristics, global optimization, stochastic optimization, etc.

PAGE TEN ▶

call for PAPERS

• Issues related to the development, implementation and evaluation of parallel optimization methods: load balancing, data structures, performance measures, software libraries, etc.

• Applications: transportation, telecommunication, location, manufacturing, TSP/VRP, finance, biology and chemistry, etc.

To submit a paper, one may send the manuscript to one of the editors either in paper form - five (5) copies are then required - or electronically as a LaTex file. A specific LaTex Style File for the Annals is available either from the editors or directly from the home page of Baltzer Science Publishers: http://www.nl.net/~baltzer/ . Electronic submissions are encouraged. As an additional incentive, the publisher offers 50 instead of the normal 25 free reprints when the authors use the Baltzer LaTex Style File.

The manuscript must be original, previously unpublished and not currently under review for other journals. To be considered for the special issue, the manuscript must be received by April 15, 1996. All manuscripts will be strictly refereed according to the highest standards as outlined in the guidelines of the Annals of Operations Research. For further information, please contact either of the guest editors below:

Teodor Gabriel Crainic
Centre de recherche sur les transports
Universite de Montréal
C.P. 6128, succ. Centre-ville
Montréal (QC) Canada H3C 3J7
(514) 343 7143
fax: (514) 343 7121
theo@crt.umontreal.ca

or

Catherine Roucairol
Laboratoire PRISM
Universite de Versailles
45, Avenue des Etats-Unis
78035 Versailles Cedex, France
(+33 1) 39 25 40 88
fax: (+33 1) 39 25 40 57
Catherine.Roucairol@prism.uvsq.fr

ISORA

Second International Symposium on Operations Research and its Applications (ISORA '96)

Guilin, China

December 11-13, 1996

The International Symposium on Operations Research and its Applications is a forum for scientists, engineers, and practitioners throughout the world to exchange ideas and research results related to operations research and its applications. The first symposium (ISORA '95) was held in Beijing, China, in August, 1995. The second symposium (ISORA '96) will be held December 11-13, 1996, in Guilin, China.

Topics of interest include but are not limited to: Linear and nonlinear programming, Combinatorial and global optimization, Multiobjective optimization, Stochastic programming, Scheduling and network flow, Queuing systems, Quality technology and reliability, Simulation, Optimizations in VLSI, Neural network, Financial modeling and analysis, Manpower planning, Production/Inventory control, Flexible manufacturing systems, Decision analysis, Decision support systems, Micro-computer software of OR methods. Papers on real-world applications will be especially appreciated.

The symposium is sponsored by the Asian-Pacific Operations Research Center within the Association of Asian-Pacific Operational Research Societies (APORS) and Chinese Academy of Sciences (CAS).

The symposium chair is Professor Xiang-Sun Zhang (CAS).

The ISORA '96 is supported by the Institute of Applied Mathematics, Chinese Academy of Sciences; the Operations Research Society of China; the National Natural Science Foundation of China; and the State Science and Technology Commission of China.

The ISORA '96 will be held in Guilin, a city in subtropical south China with a reputation for having the country's most beautiful scenery.

Authors are requested to submit five copies of an extended abstract as follows:

Organizing Committee Chair:
Prof. Kan Cheng, Institute of Applied Mathematics, Chinese Academy of Sciences, Beijing 100080, P.R. China.
fax: 86-10-254-1689
isora@amath3.amt.ac.cn

or

Program Committee Chair:
Prof. Ding-Zhu Du, Computer Science Department, University of Minnesota, Minneapolis, MN 55455, U.S.A.
fax: 612-625-0572
dzd@cs.umn.edu

Submissions should be written in English, at most ten pages, and include the e-mail address of one author who is responsible for all correspondence. One author of each accepted paper should attend the conference and present the paper. Proceedings of ISORA '96 will be published by Beijing World Publishing Corporation, and selected papers will be put in a special issue of the *Journal of Global Optimization*.

The conference welcomes any special session on the above topics. Proposals for special sessions should be sent to one of the addresses above before **July 1, 1996**.

Deadline for submission of papers:
June 1, 1996

Notification of acceptance:
August 1, 1996

Camera-ready manuscript due:
September 1, 1996

For information about program, registration and local arrangements, please contact:
Dr. X.-D. Hu
fax: 86-10-254-1689
ISORA@amath3.amt.ac.cn

or Prof. Ding-Zhu Du
fax: 612-625-0572
dzd@cs.umn.edu

REPORT ON THE THIRD WORKSHOP ON GLOBAL OPTIMIZATION

Szeged, Hungary

December 1995

OPTIMA

The Third Workshop on Global Optimization was held in December 1995 in Szeged, Hungary, and was organized by the Austrian and the Hungarian Operations Research Societies. More than 60 participants followed a tight schedule of 45 talks. The papers covered many aspects of the field, such as new heuristics, utilization of structural information, methodological questions, complexity, efficiency and reliability of global optimization algorithms. The reported applications dealt with such diverse subjects as protein folding, financial problems, tracking elementary particles, chemical process network synthesis, water quality management, optimal rejuvenation policy, and reconstruction problems in picture processing.

Participants arrived from 19 countries, including New Zealand, Jordan, Australia, Mongolia, the United States, Germany and Russia. Due to the assistance of the sponsors (Hungarian Research Fund OTKA, Veszprem University, Szeged City Mayor's Office, Pick Szeged Rt. and MOL Oilindustrial Trust), many important representatives in the field were able to participate.

The papers arising from the talks presented at the workshop will be refereed and then published by Kluwer Academic Publishers in two special issues: *Journal of Global Optimization*, and in the book *Developments in Global Optimization* in the series *Nonconvex Optimization and its Application*.

Although the five days were quite full, we did find time to see a folk dance show and to have a short sightseeing tour. There was also a reception given by the Rector of the Jozsef Attila University and another by the Mayor of Szeged. Interestingly, the Mayor of Szeged, Dr. Istvan Szalay, is a mathematician working on approximation theory.

The volume of extended abstracts, photos and many other documents of the workshop are available on the Internet at the following addresses:

URL: http://www.inf.u-szeged.hu/~globopt

FTP: ftp.jate.u-szeged.hu

in the directory /pub/math/optimization/globopt

-TIBOR CSENDES

csendes@inf.u-szeged.hu

http://www.inf.u-szeged.hu/~csendes/

NETWORK OPTIMIZATION

Report on the Conference

Gainesville, Florida

A conference on Network Optimization was held at the Center for Applied Optimization at the University of Florida, Feb. 12-14, 1996. This conference was sponsored by the National Science Foundation and endorsed by SIAM, the Mathematical Programming Society and the Institute for Operations Research and Management Science. Organizers were Panos Pardalos, Don Hearn and Bill Hager.

The conference opened with a lecture by Thomas Magnanti (MIT) on "Designing Survivable Networks." Often a network needs to be able to withstand disruptions (link or node failures) and yet still provide service to its customers. This can be achieved by building redundancy or spare capacity into the network or using different types of links (or nodes) with more reliable links connecting more essential customers. The speaker described optimization models for these situations and computational experience in solving large-scale problems with hundreds of nodes.

The speakers from nine countries discussed diverse applications in fields such as engineering, computer science, operations research, transportation, telecommunications, manufacturing, and airline scheduling. Since researchers in network optimization come from many different areas, the conference provided a unique opportunity for the cross-disciplinary exchange of recent research advances as well as a foundation for joint research cooperation and a stimulus for future research. To give an idea of the topics discussed, a few are briefly described below.

Anna Nagurney (University of Massachusetts) discussed "Massively Parallel Computation of Dynamic Traffic Problems Modeled as Projected Dynamical Systems." Computational results on the CM-5 and IBM SP2 on several traffic network examples were reported.

Warren Powell (Princeton) gave a talk on "Approximations for Multistage Stochastic Networks" in which he discussed recent results of his work which arose out of dynamic resource allocation problems.

Michael Florian and Denis Lebeuf (Montreal) presented an efficient implementation of the network simplex method which uses an extended Predecessor Index (XPI) data structure and a metaheuristic for the choice of pivot.

Donald Ratliff (Georgia Tech) and Cynthia Barnhart (MIT) presented their work on "Submodular Network Design Problems." These problems generally involve opening a subset of network elements (nodes, arcs or paths) from some larger candidate set.

Dimitri Bertsekas (MIT) reported on recent algorithmic and implementation developments using a C version of the RELAX code and standard network test problems as well as initialization techniques based on a recently proposed auction/sequential shortest path algorithm.

Robert R. Meyer (University of Wisconsin-Madison) reported on "Optimal Equi-Partition: Billion Variable Quadratic Assignment Problems." He presented an efficient method for assigning the cells of a uniform grid among an arbitrary number of processors so that load balancing constraints are observed while minimizing the overall perimeter of the partition.

Michael Grigoriadis (Rutgers) analyzed the complexity of fast approximation schemes for problems characterized by a number of disjoint convex (blocks) and a number of block-separable nonnegative convex (coupling constraints).

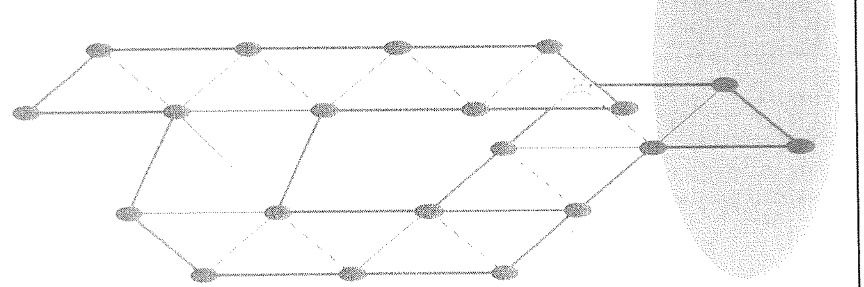
Michael Ball (University of Maryland) gave a talk on "The Rural Postman vs. the Traveling Salesman: Modeling Problems on the Border Between Arc Routing and Node Routing." He formulated several complex routing models based on actual experience with the application areas of meter reading, mail delivery and refuse pickup.

Other invited speakers included: Karen Aardal, Ronald Armstrong, John Birge, Rainer Burkhard, Narsingh Deo, Antonio Frangioni, Robert Freund, Steven Gabriel, Alexi Gaivoronski, Jean-Louis Goffin, Andrew Goldberg, Chi-Geun Han, M. Joborn, Jeffrey Kennington, Bruce Lamar, P.O. Lindberg, Athanasios Migdalas, Michael Patriksson, Rekha Pillai, Lazaros Polymenakos, Aubrey Poore, Motakuri Ramana, Ph.L. Toint, Theodore Trafalis, and Guoliang Xue.

Several participants were from industry and national laboratories including IBM Watson Research Center, AT&T Bell Laboratories, Argonne National Laboratory, NEC Research Institute, ORNL Oak Ridge, CAPS Logistics, ITALTEL Italy, and ICF Kaiser International.

Proceedings of the conference will be published later this year by Springer-Verlag.

-PANOS M. PARDALOS



OPTIMA

IN MEMORIAM

Harlan Mills

1920 - 1996

Harlan D. Mills died at his residence in Vero Beach, Florida, on January 8, 1996, at the age of 76. Harlan received his Ph.D. in Mathematics at Iowa State. He served on the faculties of Iowa State, Princeton University, New York University, University of Maryland, University of Florida, and was Professor of Computer Science at the Florida Institute of Technology. In recent years, Harlan was recognized for his brilliant work in software development (chief programmer teams, top-down design, structured programming and cleanroom software engineering), but he began his career working in mathematical programming and operations research. His paper "Marginal Values of Matrix Games and Linear Programs" (pp. 183-193 of *Linear Inequalities and Related Systems*, H.W. Kuhn and A.W. Tucker, eds., Princeton University Press, 1956) was one of the first to investigate this area.

He was one of the founders and President of the Princeton-based consulting firm Mathematica, Inc. He worked for IBM and was an IBM Fellow and a member of its Corporate Technical Committee, a technical staff member of RCA and GE, and President of Software Engineering Technology. In 1986, he was Chairman of the Computer Science Panel for the U.S. Air Force Scientific Advisory Board; from 1980-83, he was Governor of the IEEE Computer Society; and from 1974-77, he was Chairman of the NSF Computer Science Research Panel on Software Methodology. Harlan epitomized the rare scientist who knew how to integrate the ideas and methods of computer science, mathematics and operations research. All of us have been influenced and have benefited from his productive career.

-SAUL GASS

Steven Vajda

1901 - 1995

Steven Vajda, one of mathematical programming's true pioneers, passed away after a short illness on December 10, 1995.

Born in Budapest in 1901, he studied mathematics primarily in Vienna with shorter visits paid to Berlin and Göttingen, obtaining degrees in actuarial science and mathematics. After qualifying, he worked as an actuary in Hungary, Romania and Austria. In 1939, just before the outbreak of World War II, he moved to England. Like many others arriving from continental Europe at that time, Steven Vajda was interned for six months on the Isle of Man where he taught mathematics and participated in establishing a "do-it-yourself" university. During most of World War II he worked for an insurance company at Epsom but in 1944 was invited to join the British Admiralty as a statistician, soon rising to Assistant Director of Physical Research and later of Operational Research. In 1952 he was promoted to Head of Mathematics Group at the Admiralty Research Laboratory. Pat Rivett was the first Professor of OR in the UK (Lancaster University, 1963). Steven Vajda became the second when he joined Birmingham University in 1965, a position he held until his retirement in 1968 when he became a Fellow. In 1967 he was invited by Sussex University to become a Fellow and in 1973 became Visiting Professor of Mathematics, in which role he continued actively, teaching and writing research papers, for about 22 years, a record which is unsurpassed in the UK and probably anywhere outside the UK as well.

Vajda was awarded an honorary degree (D.Tech. h.c.) by Brunel University. His eminence was also recognized by the Operational Research Society (ORS) in the award of its Silver Medal, followed in 1995 by a Companionship.

Debts to Steven Vajda are in one way or another owed by many. After joining the Admiralty, he spent about 50 years consciously or unconsciously motivating the careers of numerous OR workers. He exerted this influence directly by teaching and conference presentations and indirectly by his writings and by the example of his life.

In appreciation, a group of friends and colleagues joined forces and suggested to the Mathematical Programming Study Group, ORS, that a special meeting should be organised to celebrate his work as the true founding father of mathematical programming in the UK. Focusing on duality, the meeting was eventually held in London on 10 February 1995. Among the highlights were the award of the Companionship of ORS and the warm speeches delivered afterwards at the dinner. The influence of mathematical programming was acknowledged and an early volume (Vajda, 1956) was recalled as the very first book in Europe on linear programming, being translated into French, German, Japanese, and Russian. It is indeed Steven Vajda who could rightly claim to have introduced the subject to both Europe and Asia. A report on the festive 10 February meeting appeared shortly after (Simons, 1995). Another visible outcome is the forthcoming *Special Edition of Journal of Mathematics in Business and Industry* edited by S. Powell and H.P. Williams (Powell and Williams, 1996).

Vajda's fifteenth book, *A Mathematical Kaleidoscope*, co-authored by Emeritus Professor Brian Conolly, came out just a few weeks before his death. The biographical data above is based on the section "About our Authors" found therein, on conversations with Professor B. Conolly and L.B. Kovács, on an interview in OR Newsletter (Bather, 1995), and on the citation prepared by Professor Maurice Shutler for the Award of the Companionship of the Operational Research Society to Steven Vajda (Shutler, 1995).

It is a gift of grace to enjoy a long life without suffering the physical horrors of old age and even more so to preserve both a warm heart and a brilliant mind to the end. Those gifts were granted to Professor Steven Vajda.

-JAKOB KRARUP
krarup@diku.dk

References:

Bather, J., "An interview with Steven Vajda," OR Newsletter, January 1995.

Conolly, B., and S. Vajda, *A Mathematical Kaleidoscope*, Albion Publishing, Chichester, 1995.

Krarup, J., and S. Vajda (1996a), "On Torricelli's geometrical solution to a problem of Fermat," to appear in (Powell and Williams, 1996).

Krarup, J., and S. Vajda (1996b), "Visualizing duality," submitted for publication.

Powell, S., and H.P. Williams, Eds. (1996), *Special Edition of Journal of Mathematics in Business and Industry: Duality: In celebration of work of S. Vajda.*

Shutler, M.F. (1995), "Steven Vajda," citation for the Companionship of ORS, to appear in (Powell and Williams, 1996).

Simon, R., "Founding father of British LP is honoured. Celebration of Steven Vajda's work," OR Newsletter, April 1995.

Vajda, S., *Theory of Games and Linear Programming*, Methuen, 1956.

Vajda, S., *Mathematical Programming*, Addison-Wesley, 1961.

Svata Poljak

1951-1995

Svatopluk Poljak died on April 2, 1995, at age 44 in an auto accident near Prague. He is survived by his wife Jana and their two sons Honza and Vitek.

He was born in Prague on October 9, 1951, and did his training at Charles University in Prague. He received his RNDr diploma (doctorate in natural sciences) in 1976 and obtained his PhD in 1980 under the supervision of Zdenek Hedrlin. Svata taught at the Czech Technical University in Prague from 1979 to 1986, when he rejoined Charles University as a senior researcher. He was awarded a qualification degree from the Czechoslovak Academy of Sciences in 1990. In April 1994 Svata moved to the University of Passau to take up a position on the Faculty of Mathematics and Computer Science.

Svata's contributions are best demonstrated by over 95 publications in diverse areas of combinatorics and discrete optimization. At the time of his death he was working intensively on solving discrete optimization problems such as the max-cut and stable set problems, using eigenvalue techniques and nonlinear programming approaches. The fast approximation algorithms for finding a maximum cut in a graph that recently came in the spotlight after the breakthrough paper by Michel Goemans and David Williamson, find their root in work of Svata (with Charles Delorme) on eigenvalue methods for graph problems. Indeed, the two approaches of Michel and David and of Charles and Svata are, in fact, dual in the sense of semidefinite programming duality. Svata and Charles conjectured in 1993 that the bound provided by this approximation is very close to the true optimum (within 13 percent); Goemans and Williams succeeded in proving an estimation slightly larger than the conjectured one. These results are very interesting from a theoretical point of view; moreover, they are promising for the practical purpose of solving max-cut problems, as these can now be tackled via interior point methods.

It would be difficult to summarize all of the significant contributions Svata made. One area where his work had a big impact, though perhaps not so well-known, was in neural networks. He (along with Dan Turzik) found an elegant solution to an open question concerning the periodical behaviour of finite automata.

Svata collaborated with many people throughout the world, always bringing warmth and friendship in work relationships. Many of us have lost in him a precious collaborator and a dear friend. We all miss him very much.

-MONIQUE LAURENT
laurent@dmi.ens.fr

-HENRY WOLKOWICZ

A list of Svata's publications as well as several of his publications have been posted by Henry Wolkowicz and can be found on the WWW > at URL:\\> http://orion.uwaterloo.ca/~hwolkowi/.preprints/authors.d/poljak.d/

Council news

The council has decided on the locations and the chairs of the Organizing and Program Committees of the next two IPCO-meetings. They are as follows:

IPCO 6, 1998

Houston, TX, U.S.A.
Chair of the Program Committee: Robert E. Bixby
Chair of the Organizing Committee: E. Andrew Boyd

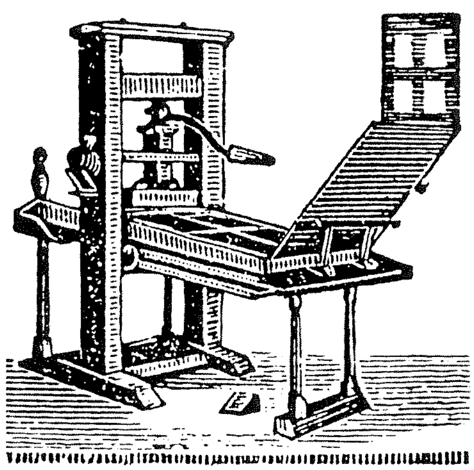
IPCO 7, 1999

Graz, Austria
Chair of the Program Committee: Gérard Cornuejols
Chair of the Organizing Committee: Rainer E. Burkard

-KAREN AARDAL

OPTIMA

BOOK REVIEWS



Nonlinear Programming

By O.L. Mangasarian
Classics in Applied Mathematics 10, SIAM
Philadelphia, 1994

ISBN 0-89876-341-2

Numerical optimization (nonlinear programming) is a rich and practical subfield of applied mathematics. Underpinning the methods of (constrained) numerical optimization is an elegant theory connecting convexity, optimality conditions, duality, and various aspects of nonlinearity. A serious student, or user, of optimization should study this theory and have the important concepts readily available. The best source over the last two decades, in terms of a healthy mix of rigor, brevity, and accessibility, has been Olvi Mangasarian's *Nonlinear Programming*. Happily, SIAM has chosen to reprint this book in its Classics series.

Nonlinear Programming has been an active member of my bookshelf for over 15 years. On occasions too numerous to count, I have reached for my slim blue copy to review a convexity result or to check a topological definition. I am usually rewarded, because this book has a remarkable quality: the important theoretical concepts of constrained optimization are there, easy to find, easy to understand.

Nonlinear Programming begins in Chapters 1-4 with basic definitions, a discussion of "theorems of the alternative" and an introduction to the fundamentals of convexity. Chapter 5 is concerned with optimality conditions without assuming differentiability. I like this chapter for two reasons. First, the material is important but not commonly presented (typically, at least differentiability is assumed). Second, the author does an excellent job of clearly indicating when convexity is required and when it is not.

Chapter 6 discusses differentiable convex and concave functions; Chapter 7 presents well-used optimality conditions; Chapter 8 discusses duality (a topic that has played a prominent algorithmic role in recent years). The presentation here is distinguished by simple diagrams to illustrate key points, e.g., various constraint qualifications and, most importantly, diagrams that graphically illustrate the relationships amongst various problems/optimality characterizations.

Generalizations of convex functions - quasiconvex and pseudoconvex functions - are discussed in Chapters 9 and 10. The hallmark of these chapters is that subtle relationships are exposed in simple diagrammatic form.

Chapter 11 adds nonlinear *equality* constraints to the mix. Optimality and duality results are re-examined in this new light. Four appendices, following Chapter 11, form an important part of this book: Relevant topics from linear algebra, topology, and real analysis are summarized. These appendices contain important background material for the main part of the book. Moreover, they represent a convenient packaging of basic mathematics used in optimization.

In conclusion, *Nonlinear Programming* is indeed a classic. I fully agree with the author, "it is a concise, rigorous, yet accessible account of fundamentals of constrained optimization theory that is useful both to the beginning student as well as the active researcher." This is a resource that can benefit every serious "student" of optimization.

It is important to understand what a book is not, as well as what it is. This book is not about algorithms or methods (none are discussed). It is certainly not about practical computing issues. It is not concerned with complexity issues. It is also difficult for me to imagine this book as a primary text for a course, except perhaps a course taught by an expert in this area who could "smooth out" the material with some examples of applicability and additional motivating material. The value of this book is as a resource: it is a wonderful summary of important supporting theory for nonlinear optimization, especially with respect to constraints.

-THOMAS F. COLEMAN

Control and Optimization

by B.D. Craven
Chapman and Hall
London, 1995

ISBN 0-412-55890-4

This volume treats control problems governed by ordinary differential and difference equations from a unified standpoint of optimization in normed vector spaces. There are a vast number of articles in scientific journals on this topic, but no comprehensive monograph has previously appeared. Therefore, this book is a first step in this direction.

The ideas of optimization are introduced in the first chapter by means of some simple examples in finite and infinite dimensions. Some mathematical background is also presented spanning such diverse material as simple matrix calculations and the rather deep idea of weak compactness and Alaoglu's theorem. Basic measure theory is summarized on one page.

Chapter Two describes six basic dynamic control models and discusses the respective cost functions to be optimized: advertising models, investment models, production and inventory models, water management models, fish population models, and epidemic models. Since the 1950s, optimal control theory has been so influenced by spacecraft problems such as fuel minimization and trajectory accuracy, it is astonishing that a detailed example of this technical type is not presented here.

Chapters Three and Four form the core of the book. Chapter Three gives a balanced introduction to convexity, linearization and multipliers in abstract spaces. Topics are separation properties, theorems of the alternative, convex and invex functions, Karush-Kuhn-Tucker conditions, (quasi) duality, and an outlook on nondifferentiable optimization. On this basis, optimality conditions for control problems are derived in Chapter Four, both for discrete and continuous time problems. The main result is the Pontryagin maximum principle, a detailed proof of which is delayed until Chapter Seven. Some worked problems exemplify the ideas. Time optimality and sensitivity and stability are discussed as well. Only first order necessary conditions are presented; the reader does not find any material on second order conditions.

Chapter Five elaborates on the worked examples introduced in Chapter Two. The concept of singular arcs in the case where the control appears linearly in the model is discussed.

Chapter Six is dedicated to algorithms for optimal control problems. There have been some interesting extensions of finite dimensional optimization algorithms (Newton-, quasi-Newton- and conjugate gradient algorithms) to infinite dimensions, but these algorithms are neither efficient in their computational realization nor are they implemented in robust professional-like packages. (The state of the art in computational optimal control, direct and indirect methods, shooting, finite difference and collocation techniques, which are the natural approaches for differential equations as side constraints, is summarized in [1].)

Except for the many misprints in mathematical formulae, where vectors are not typed in boldface, which greatly hinders the readability of the text, this book can be warmly recommended to mathematicians who are interested in a quick introduction to the subject. It is not equally worthwhile for engineers interested in a more heuristic approach.

[1] Burlirsch, R., D. Kraft: ***Computational Optimal Control*****. Birkhäuser, Boston, 1994.**

-DIETER KRAFT

Application for Membership

I wish to enroll as a member of the Society.

My subscription is for my personal use and not for the benefit of any library or institution.

☐ *I will pay my membership dues on receipt of your invoice.*

☐ *I wish to pay by creditcard (Master/Euro or Visa).*

CREDITCARD NUMBER: ______________________ EXPIRY DATE: ______________

FAMILY NAME: ______________________

MAILING ADDRESS: ______________________

____

TEL.NO.: ______________________ TELEFAX: ______________

E-MAIL: ______________________

SIGNATURE ______________________

Mail to:

The Mathematical Programming Society, Inc.
c/o International Statistical Institute
428 Prinses Beatrixlaan
2270 AZ Voorburg
The Netherlands

Cheques or money orders should be made payable to The Mathematical Programming Society, Inc., in one of the currencies listed below.
Dues for 1996, including subscription to the journal *Mathematical Programming*, are Dfl.105.00 (or \$60.00 or DM94.00 or £39.00 or FF326.00 or Sw.Fr.80.00).
Student applications: Dues are one-half the above rates. Have a faculty member verify your student status and send application with dues to above address.

Faculty verifying status

____

institution

____

Donald W. Hearn, EDITOR
hearn@ise.ufl.edu

Karen Aardal, FEATURES EDITOR
Utrecht University
Department of Computer Science
P.O. Box 80089
3508 TB Utrecht
The Netherlands
aardal@cs.ruu.nl

Faiz Al-Khayyal, SOFTWARE & COMPUTATION EDITOR
Georgia Tech
Industrial and Systems Engineering
Atlanta, GA 30332-0205
faiz@isye.gatech.edu

Dolf Talman, BOOK REVIEW EDITOR
Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands
talman@kub.nl

Elsa Drake, DESIGNER

PUBLISHED BY THE
MATHEMATICAL PROGRAMMING SOCIETY &
GATOREngineering® PUBLICATION SERVICES
UNIVERSITY OF FLORIDA

Journal contents are subject to change by the publisher.

gallimaufry

number 49
MARCH 1996

Remember that preregistration for ISMP97 is via World Wide Web: http://dmawww.epfl.ch/roso.mosaic/ismp97/welcome.html The e-mail is: ismp97@masg1.epfl.ch

¶Deadline for the next OPTIMA is May 15, 1996.

O P T I M A
MATHEMATICAL PROGRAMMING SOCIETY

UNIVERSITY OF FLORIDA

Center for Applied Optimization
371 Weil Hall
PO Box 116595
Gainesville FL 32611-6595 USA

FIRST CLASS MAIL